FOLIATION GEOMETRY/TOPOLOGY PROBLEM SET

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Contents

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1. Introduction

This is a collection of problems and related comments concerning the geometry, topology and classification of smooth foliations. It was prepared in anticipation of the "golden month" of meetings on foliations in Japan during September 2003, and updated following the meetings with some additional questions, and various corrections. At the end of the list of problems is a very extensive (and excessive, but useful) set of references related to the questions raised.

"Foliation problem sets" have a long tradition in the study of this subject – they highlight progress in areas of research, and hopes for progress. Smale's celebrated survey in 1967 of dynamical systems [540] might be considered the first foliation problem set, as many of the questions about dynamical systems lead to questions about the properties of foliations associated to dynamical system.

Foliation theory had its own seminal survey, given by H. Blaine Lawson in the Bulletin of the AMS in 1974 [336] at the beginning of a period of very rapid development in the field. Lawson's subsequent lectures at Washington University in St. Louis in 1975 [337] captured the spirit of progress in the subject at that time, especially the very recently proven foliation existence theorems by Thurston for higher codimensions [572] and codimension one [573], and the works of many authors on the existence, properties and evaluation of secondary classes.

The year 1976 was a critical year for conferences reporting on new results in foliation theory – in particular, there were major conferences with problem sessions on foliations at Stanford (compiled by Mark Mostow and Paul Schweitzer [431]) and at Rio de Janeiro (compiled by Paul Schweitzer [514]). These problem sessions posed questions which were prescient, foreshadowing developments in the subject through the 1980's and 90's – yet many of the problems raised still remain open.

The meeting "Differential Topology, Foliations and Group Actions. Rio de Janeiro 1992", included an extensive collection of problems compiled by Remi Langevin [328]). Notable is how many of the suggested problems were unchanged during the 16 years between these two conferences.

The meeting "Analysis and geometry in foliated manifolds" in Santiago do Compostela 1994 included a short problem set at the end of its proceedings [362].

The Seminaire Bourbaki report "Sur l'invariant de Godbillon-Vey" by Étienne Ghys [184] included a number of problems with discussions about them. The author's survey of the Godbillon-Vey class [261] in the proceedings of the conference "Foliations: Geometry and Dynamics (Warsaw, 2000)" reported on further developments in the area of foliation dynamics and secondary classes.

This problem set was begun for inclusion in the proceedings of the Warsaw 2000 conference.

The field of foliations becomes broader and more deeply explored with each decade – it is now more than fifty years since Reeb's original paper [488]! To get some perspective, consider that the first five sections of Lawson's 1974 survey was dedicated to results and questions about the existence of foliations on particular manifolds, and reported on examples and methods of construction of foliations due to wide variety of authors, including A'Campo [1], Alexander [8], Arraut [11], Durfee [129, 130], Fukui [158], Lawson [130, 335, 498], Laudenbach [334], Lickorish [357], Moussu [432, 434], Novikov [447, 448, 449, 450], Reeb [488, 489, 490], Reinhart [393], Rosenberg [496, 497, 499, 500, 501, 503, 504], Roussarie [499, 500, 501, 198, 503, 505, 506], Schweitzer [513], Tamura [564, 565, 566, 567, 406, 407, 408], Thurston [570, 504], Wood [632, 633, 634, 635] to mention some.

The last sections of Lawson's article cited the developing theory of general existence and non-existence results by Bott [41, 42, 43, 52], Gromov [201], Haefliger [210, 211, 212, 213], Phillips [468, 469, 470], and Thurston [571, 572, 573].

Finally, Lawson's survey was written just at the advent of "classification theory" for foliations in the 1970's and the extremely powerful ideas and techniques of Gromov [201], Haefliger [212, 213], Mather [364, 365, 366, 367, 368, 369], McDuff [377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390], Phillips [468, 469, 470], Segal [517, 518, 519, 390, 520], and Thurston [571, 572, 573], led to an increased emphasis on the "machinery" of algebraic and differential topology in the subject, and with less prominence given to developing the techniques of construction.

It is correspondingly more difficult to survey the many developments in the field, and to compile and comment on problem sets across the variety of active topics of research. The focus of this problem set is thus more restricted than we might hope for. For example, taut foliations on 3-manifolds was one of the main topics of the Warsaw conference, and has been an extremely active area of study recently. There is a comprehensive, up-to-date set of problems on this topic, compiled by Danny Calegari, and available on the web at [68]. The omission of the many important questions about foliations on 3-manifolds from this problem set is thus justified.

However, there are many other topics, which are unfortunately not covered either, including: "rigidity and deformations of foliations", "Riemannian foliations", "Riemannian geometry of foliated manifolds", "holomorphic foliations", "singular foliations" , "confoliations and contact structures", "analysis of foliated manifolds", "index theory and cyclic cohomology of foliations", and "Gel'fand-Fuks cohomology, cyclic cohomology and Hopf algebras" are not covered. The list of topics not covered shows how incredibly far the subject has developed since Reeb's Thesis!

The problems included here are posed by a variety of researchers. Some of the problems have been previously published, and we include follow-up comments were possible. Other problems are from the author's personal collection of mathematics problems encountered over the last 25 years. Where known, the names of the person who suggested a problem is included, and where the problem has appeared previously is cited. The author has attempted to include only those problems where there is some feeling that the current status can be accurately reported. In any case, misstatements, misunderstandings and outright fallacies concerning any of these problems are all due to the author.

Two years late for the Warsaw deadline, these problems were prepared for and distributed during the meetings "Geometry and Foliations 2003" at Ryukoku University, Kyoto, and "BGamma School – Homotopy Theory of Foliations" at Chuo University, Tokyo during September and October 2003. Comments, additions and corrections to this revised version of the problem set reflect comments offered during these meetings.

On a personal note, the author's impression is that given the current state of research into classification theory of foliations, described in the last sections, to make progress now demands new techniques and ideas combining geometric methods with the algebraic and classifying space classification methods. We include in particular a number of problems which highlight constructions that would be interesting to know how to do. This focus on "building foliations" is an argument for "returning to its roots" – a reading of the literature cited at the end shows how many papers on foliations in the 1960's and 1970's were dedicated to geometric constructions. One underlying theme of this problem set is that all of the technological developments of the thirty years can now serve as a guide for new constructions.

2. Geometry of leaves

There is a long tradition of questions about which open complete manifolds can be the leaf of a smooth foliation of a compact manifold. This article by Sondow [542] was perhaps the first time this question appeared in print, but it is one of the most natural questions about foliations. This is called the "realization" problem, and it is one of the more difficult problems to study, as there are only a small collection of techniques available, which have yielded sporadic results – both positive and negative. We first fix the notation and context.

Let M be a compact manifold with a C^r foliation $\mathcal{F}$ of leaf dimension p and codimension q. A choice of a Riemannian metric g on TM determines a Riemannian metric g_L on each leaf $L \subset M$ of $\mathcal{F}$. The pair (L, g_L) is Riemannian manifold with the geodesic length metric $d_L \colon L \times L \to [0, \infty)$. Then (L, d_L) is a complete metric space with a unique quasi-isometry type. A more precise version of the realization problem, is to ask what restrictions are placed on the Riemannian geometry of a leaf of a foliation, and what restrictions on the quasi-isometry type are imposed by the hypotheses that a manifold has the distance function derived from such a metric. In short, what are the restrictions on the intrinsic geometry of a leaf?

The problem of realizing surfaces as leaves in 3-manifolds has the oldest tradition, starting with the examples in Reeb's thesis [488]. The first systematic treatment was made by W. Bouma and G. Hector, who showed in [58] that every open surface can be the leaf of a codimension one smooth foliation of $\mathbb{R}^3$. Cantwell and Conlon then showed in [79] that every open surface can be a leaf of a codimension one smooth foliation of a *compact* 3-manifold. In neither case, do the authors make any assertion about the quasi-isometry types which can be realized. In contrast, there are several constructions of quasi-isometry classes of surfaces which cannot be realized, given by Abdelghani Zeghib [638] and Paul Schweitzer [516]. The obstruction to realization uses the concept of the leaf entropy, introduced by Oliver Attie and the author [18, 257].

An open surface is planar if it can be embedded in $\mathbb{R}^2$. In other words, the manifold is diffeomorphic to $\mathbb{R}^2 - K$ where K is some closed subset. For example, K might be a closed subset of the interval $[0, 1]$, and so inherits topological properties such as the sequence of derived sets associated to K. Given any countable ordinal β, it is possible to construct a closed subset $K_\beta \subset [0, 1]$ whose derived sequence corresponds to β. This result is most likely due to G. Cantor, but the argument was shown to the author by Dave Marker [361].

PROBLEM 2.1. *For any countable ordinal β, give a constructive procedure for producing a foliated compact manifold with a leaf L such that the endset $\varepsilon(L)$ has derived series α. For example, how do you realize a leaf corresponding to the first exceptional ordinal $\epsilon_0 = \omega^{\omega^{\omega^{\cdot^{\cdot^{\cdot}}}}}$?*

The indirect way to do this is to first realize ϵ_0 as a closed subset of $\mathbb{R}^2$, then use the construction of Cantwell and Conlon [79] to realize the complement of this set as a leaf. The underlying point of the question is whether the finitely generated pseudogroup structure of a foliation imposes any order properties on the endsets of the leaves, like an automatic group structure, which can then be related to the logical properties of the endsets (cf. the work of Adams and Kechris [6]). This would seem to be an area of foliation or pseudogroup theory where little has been proven, while the analogous topics for groups is well-developed (see [189, 190] for just an introduction to this topic – the literature on geometric group theory is vast.)

For foliations with leaves of dimension 3, the realization question has a much different answer. In 1985, both Ghys [177] and T. Inaba, T. Nishimori, M. Takamura and N. Tsuchiya [287] showed there are 3-dimensional open manifolds which are not homeomorphic to leaves of any codimension one foliation of a compact manifold. The open 3-manifolds which these authors construct have completely aperiodic topology on one of their ends. There is, as yet, no general understanding of which 3-manifolds can be realized as leaves.

PROBLEM 2.2. *Which open 3-manifolds are homeomorphic to some leaf of a foliation of a compact manifold?*

A *Whitehead manifold* W is an irreducible, contractible open 3-manifold which is not homeomorphic to $\mathbb{R}^3$. The original Whitehead manifold W (described in Rolfson, page 82, [495]) is an end-periodic 3-manifold. Paul Schweitzer observed that W can be a leaf in a homological Reeb component in codimension one, since it is obtained from an embedding $\phi : C \to Int(C)$, where C is the solid torus, as the direct limit of ϕ iterated repeatedly.

The following problem was proposed by Schweitzer in the meeting in Kyoto:

PROBLEM 2.3. *Can every Whitehead manifold W be a leaf of a codimension-one foliation of a compact 4-manifold? Of a foliation of any codimension?*

The point of the question is that there are Whitehead manifolds defined by a sequence of embeddings $\phi_n : C \to Int(C)$, where $W = \lim\limits_{n\to\infty} (C, \phi_n)$ and the ϕ_n's are not eventually periodic. One could try to first show that if such a W is a proper leaf, then the end of the leaf has some strong periodic limiting behavior, and hence the ϕ_n are eventually periodic, contrary to assumption. If such a W is embedded as a non-proper leaf, then the quasi-uniformity of such leaves seems to be in opposition to the construction of W. So, in both cases, there is work to do – this seems like a very good question! Moreover, this question shows the limits of our current understanding of "what leaves look like".

It is also natural to ask the same question for non-contractible 3-manifolds W obtained as above from a sequence of embeddings $\phi_n : C \to Int(C)$ that is not eventually periodic.

For foliations with leaves of dimension 4, there is an analogous question, whether it is possible to realize an exotic $\mathbb{R}^4$ as a leaf. This problem is even more interesting, due to the existence of exotic differential structures on $\mathbb{R}^4$. Recall that an exotic $\mathbb{R}^4$ is a smooth manifold X which is homeomorphic to Euclidean $\mathbb{R}^4$, but is not diffeomorphic to $\mathbb{R}^4$. There are in fact continuous families of such exotic beasts (see Gompf [199], Furuta and Ohta [164].) There are also constructions of continuous families of exotic differentiable structures of open 4-manifolds (see [36, 107]). All of these manifolds are not the coverings of a compact manifold, ad it seems just as unlikely that they could be leaves of a foliation.

PROBLEM 2.4. *Show that a smooth manifold L which is an exotic $\mathbb{R}^4$ cannot be a leaf of a C^1-foliation $\mathcal{F}$ of a compact manifold M.*

One supposes that the same sort of restriction is true for the other constructions of exotic open 4-manifolds, but the author knows of no results published on this topic.

For leaf dimensions greater than four, there are no general results about the realization problem. Restrictions on which manifolds are homeomorphic to leaves *should* exist – in the spirit of the work of [177, 287] – they just haven't been discovered yet. Here are three basic questions:

PROBLEM 2.5. *Give a complete Riemannian manifold which is not homeomorphic to a leaf of a foliation in codimension greater than one.*

The problem is really to develop new "recursion invariants" of the homotopy type of a leaf, and then construct smooth manifolds whose values for these invariants obstructs their being a leaf. For example, the Whitehead manifolds have the property thatthey are not "tame at infinity", and the conjecture above is that only the end-periodic examples can be realized as leaves. Here is a generalization of this conjecture:

PROBLEM 2.6. *Suppose that L is a smooth contractible manifold. If L is a leaf of a C^1-foliation $\mathcal{F}$ of a compact manifold M, what can be said about the end-periodic structure of L? Must L be end-periodic?*

The work of Oliver Attie (see [16, 17, 18, 257]) on surgery on open manifolds gives a way to descibe algebraicly the surgery sets for leaves, but these are formulated in terms of uniform structures, so are not yet homeomorphism invariants. One can ask for some form of algebraic classification of the surgeries that result in leaves of foliations:

PROBLEM 2.7. *Let L be an open complete n-manifold for $n \geq 5$. If L is homeomorphic to a leaf of a foliation of a compact manifold, does this imply restrictions on the set of manifolds homotopy equivalent to L?*

Now consider the more metric realization problem, where we assume L is a smooth leaf of a foliation of a compact Riemannian manifold. The problem is, what restrictions are thus placed on the Riemannian geometry of L, and what restrictions on the quasi-isometry type are imposed by the hypotheses that a manifold has the distance function derived from such a metric? In short, what are the restrictions on the intrinsic geometry of a leaf?

Let L be a complete Riemannian manifold. Fix a base-point $x \in L$, then let $Vol_L(x, R)$ denote the volume of a ball centered at $x \in L$ with radius R. Say L has:

- *subexponential growth* if $\limsup_{R\to\infty} \log\{Vol_L(x, R)\}/R = 0$.
- *non-exponential growth* if $\liminf_{R\to\infty} \log\{Vol_L(x, R)\}/R = 0$.
- *exponential growth* if $\liminf_{R\to\infty} \log\{Vol_L(x, R)\}/R > 0$.

These properties are independent of the choice of the basepoint, and are quasi-isometry invariants of the geodesic metric space L. If L is a leaf of a foliation of a compact manifold M, then the choice of a Riemannian metric on M determines a unique quasi-isometry class of metrics on L.

One can then ask what growth types occur as leaves of foliations? It was realized in the early 1970's that exponential growth types are associated with hyperbolic systems, resilient leaves and exceptional minimal sets [433, 476, 479], while subexponential growth is associated with parabolic group actions, and homology cycles [480, 481, 558] – especially through the works of Joseph Plante (see also [475, 480, 481, 482, 483].)

Phillips and Sullivan [471] and later Januszkiewicz [291] showed there are obstructions to realizing a certain quasi-isometry types of manifolds with subexponential growth type in a given closed manifold, based on the asymptotic homology cycles determined by a leaf. The obstructions they developed required the leaf to have subexponential growth in order to construct asymptotic homology cycles, and the ambient manifold to have some vanishing properties in cohomology.

Attie and Hurder [18, 257] introduced in 1993 an invariant of quasi-isometry type which gave an obstruction to a complete manifold being realized as a leaf of a foliation in any codimension, and which applied to all growth types. It is then possible to construct many examples of "exotic quasi-isometry types" - those which are highly chaotic at infinity – and prove that these cannot be leaves. The Attie–Hurder constructions of "random water towers" on open manifolds used surgeries in higher dimensions. Zeghib [638] and later Schweitzer [516] gave simplified constructions of these examples using "bubbles of positive curvature", which applied to surfaces.

There is a more refined quasi-isometry invariant of leaves, called the growth function, which is the equivalence class of the volume growth function for geodesic balls, $R \mapsto Vol_L(x, R)$.

PROBLEM 2.8. *Which classes of growth functions arise as a leaf of a C^r-foliation, for $r \geq 0$?*

The growth type of a leaf is at most exponential, but the existence of leaves with various intermediate (or fractional) growth types between polynomial and exponential may differ for the cases $r = 1, \infty, \omega$, for example. The realization of various growth types via constructions has been studied by Cantwell and Conlon [73, 76, 77], Hector [229, 230, 231, 233], Inaba [279], and Tsuchiya [594, 595, 596, 597].

Marek Badura has more recently studied the problem of realizing various recursively defined growth types as open complete manifolds, and as leaves of foliations [19, 20]. This work is in the spirit of Problem 2.1, as he uses explicit constructions to realize the growth types.

We mention one more class of problems, which are more close to rigidity questions, but are phrased in terms of the existence of prescribed Riemannian metrics on the leaves.

PROBLEM 2.9. *Can a codimension one foliation have higher rank?*

Burger and Monod [65, 66] and Ghys [188] showed that a higher rank group does not admit an effective C^1-action on the circle. One can view these results as non-existence theorems for a codimension one foliation transverse to a circle bundle with non-compact leaves. Can these theorems be generalized to codimension one foliations which are not transverse to a circle bundle? Part of the problem is to give a suitable definition of higher rank for a foliation (cf. [641, 7, 5].)

The recurrence properties of a leaf in a foliated compact manifold are a consequence of a weak version of covering transformations - the leaves admit a cocompact action of a pseudogroup, though not necessarily of a group. Based on this analogy between covering spaces and leaves, one expects:

PROBLEM 2.10. *Show that the non-standard examples of complete manifolds with negative sectional curvature constructed by Gromov and Thurston* [206] *cannot be realized as leaves of a C^1-foliation of a compact manifold.*

3. Dynamics of leaves

Let M be a compact manifold without boundary, and $\mathcal{F}$ is a C^r foliation with leaf dimension p and codimension q. The theme of the problems in this section concerns the recurrence properties of non-compact leaves in a foliation, and in what way does the dynamics of a single leaf, or collection of leaves, impose similar behavior for the dynamics of the other leaves.

We begin with a beguilingly simple question posed to the author by Scot Adams. This was problem A.3.1 in the Rio 1992 problem set [328]:

PROBLEM 3.1. *Suppose that $\mathcal{F}$ is a topological (or C^1, C^2, etc.) foliation of a compact manifold M. Is it possible that $\mathcal{F}$ has exactly one non-compact leaf, with all of the remaining leaves compact?*

The answer is no in codimension one, as the set of the compact leaves is a compact set. Elmar Vogt proved that for a topological foliation in codimension two, $\mathcal{F}$ is either a Seifert fibration, or has uncountably many leaves [617].

A foliation with at most countable number of non-compact leaves is called *almost compact* in [254]. It is known that every leaf of an almost compact foliation must be proper. If an almost compact foliation admits a cross-section (a closed transverse submanifold which intersects every leaf of the foliation) then every leaf must be compact and the foliation is a generalized Seifert fibration [254]. The current formulation of the problem is thus, does there exists a foliation of codimension greater than two on a compact manifold M with at most countable number of non-compact proper leaves, and all of the remaining leaves compact?

A foliation is said to be essentially compact if the set of non-compact leaves has Lebesgue measure zero. Tracy Payne constructed in [466] examples of essentially compact foliations, which settled Problem A.3.2 of [328]. Here is a related question:

PROBLEM 3.2 (A. Fahti, M. Herman)**.** *Does there exists a C^1-diffeomorphism of a compact manifold M such that* almost every *orbit is periodic, but the map is not of finite order? Can such a map be found which is volume-preserving?*

REMARK: The methods of Edwards, Millet and Sullivan [131] are sufficient to show (with slight modification) that if the set of non-periodic orbits does not separate the manifold M, then the map has finite order. Thus, the problem is to replace the topological non-separating condition on the non-periodic orbits, with a condition that they do not "separate in measure".

Recurrence properties of leaves are related to their growth types, though the relation is not always transparent. Gilbert Hector showed in 1977 that a foliation can have leaves which have nonexponential growth but not subexponential growth [230].

PROBLEM 3.3. *Show the set of leaves with non-exponential growth, and not subexponential growth, has Lebesgue measure zero.*

Hector's construction in [230] of examples with leaves of this special type appear to produce a set non-exponential leaves with measure zero. This growth condition, that $\limsup \neq \liminf$, implies a high degree of non-uniformity for the asymptotic limits of the leaf. If there exists a set of positive measure consisting of such leaves, then recurrence within the set should imply a uniformity of the growth, contradicting the hypothesis. At least, that is the hope.

The following question was posed by Ghys around 1994. It remains open to the author's knowledge.

PROBLEM 3.4 (E. Ghys)**.** *Does there exists a foliation $\mathcal{F}$ of a compact manifold M such that for any two leaves $L, L' \subset M$, L is homeomorphic to L' if and only if they are the same leaf? Even, can this happen in codimension-one? What is known of the cardinality of distinct (up to homeomorphism) leaves in a codimension-one foliation?*

Given a compact manifold M, the classification of the codimension-one C^1 foliations without holonomy on M is essentially completely understood. For higher codimensions, even a modest structure theorem becomes hopeless. For example, given any collection of flows without periodic orbits on manifolds $M_1, \ldots, M_k$ there is a free action of $\mathbb{R}^k$ on $M = M_1 \times \cdots M_k$. Since all orbits are contractible for this action, the foliation it defines has no holonomy.

There is another standard construction which provides examples whose leaves are not flat manifolds. Let Γ be a lattice subgroup with a dense, faithful representation $\alpha\colon \Gamma \to \mathbf{SO}(n)$ – such exists by usual methods of lattice theory [643]. Let B a manifold with fundamental group isomorphic to Γ. Then the suspension construction [69] yields a foliation $\mathcal{F}_\alpha$ on a manifold M which fibers over B with fibers $\mathbf{SO}(n)$. Then $\mathcal{F}_\alpha$ is a foliation without holonomy, whose leaves are all diffeomorphic to the universal covering $\widetilde{B}$ of B.

Even considering these examples and others, we still propose:

PROBLEM 3.5. *Let $\mathcal{F}$ be a C^1 foliation of a compact manifold M. Suppose that every leaf of $\mathcal{F}$ is without holonomy. Is there a restriction of the isometry types o leaves of $\mathcal{F}$? Can the leafwise entropy introduced in* [18] *be non-zero for such a foliation?*

4. Foliation entropy and transverse expansion

One of the most fundamental invariants of the dynamics of a diffeomorphism of a compact manifold is its topological entropy. When positive, it implies the orbits of f exhibit an exponential amount of "chaos". When zero, the map f is somehow not typical, and has unusual regularity. The corresponding entropy invariant for foliation dynamics is the geometric entropy $h_g(\mathcal{F})$ for a C^1–foliation $\mathcal{F}$ introduced by Ghys, Langevin and Walczak [193]. The geometric entropy $h_g(\mathcal{F})$ measures the exponential rate of growth for (ϵ, n)-separated sets in the analogue of the Bowen metrics for the holonomy pseudogroup $\mathcal{G}_{\mathcal{F}}$ of $\mathcal{F}$. Thus, $h_g(\mathcal{F})$ is a measure of the complexity of the transverse dynamics of $\mathcal{F}$. The precise value of $h_g(\mathcal{F})$ depends upon a variety of choices, but the property $h_g(\mathcal{F}) = 0$ or $h_g(\mathcal{F}) > 0$ is well-defined. (Chapter 13 of [71] gives an excellent introduction and discussion of foliation entropy.) There are many fundamental questions about this invariant; the author's papers [259, 260, 261] discuss recent developments in how the geometric entropy is related to transverse hyperbolicity.

PROBLEM 4.1. *Suppose $\mathcal{F}$ is a C^1 (or possibly C^2) foliation of codimension $q > 1$. What does $h_g(\mathcal{F}) > 0$ imply about recurrence properties of the leaves of $\mathcal{F}$.*

In codimension one, $h_g(\mathcal{F}) > 0$ implies $\mathcal{F}$ has a resilient leaf. In higher codimensions, the natural generalization of the resilient property for a leaf L is that there is an element of holonomy h_γ for L which is locally transversely contracting, and some end of L intersects the domain of h_γ. The hypothesis that there is a contracting element is simply too strong, so the question is also to formulate a qualitative dynamical property of a foliation which $h_g(\mathcal{F}) > 0$ forces to exists. Some criteria are given in [259], see also [252].)

Ghys, Langevin and Walczak [193] showed that $h_g(\mathcal{F}) = 0$ implies there exists a transverse invariant measure for $\mathcal{F}$. The absence of a transverse invariant measure implies that $\mathcal{F}$ has no leaves of nonexponential growth [481], but it is unknown if this suffices to imply $h_g(\mathcal{F}) > 0$.

PROBLEM 4.2. *Suppose $\mathcal{F}$ is a C^1 (or possibly C^2) foliation of codimension $q > 1$. Formulate qualitative dynamical properties of a foliation which are implied by $h_g(\mathcal{F}) > 0$, and are sufficient to imply $h_g(\mathcal{F}) > 0$.*

For example, one approach to this was discussed in the program outline [252], where the key open issue is to develop a theory of measure entropy for a foliation.

PROBLEM 4.3. *Give a definition of the* measure *entropy, or some other entropy-type invariant, of a C^1-foliation $\mathcal{F}$, which can be used to establish positive lower bounds for the geometric entropy.*

This problem was asked in the original paper of Ghys, Langevin and Walczak [193]. Their earlier paper [192] gave a possible definition, but the connection to the geometric entropy is unclear. Hurder proposed a definition of the measure entropy in terms of invariant measures for the associated geodesic flow [252]. Another approach might be to define measure entropy for a foliation in terms of its harmonic measures.

We mention also the work of Shinji Egashira [132, 133, 134], who developed the concept of the expansion growth rate function, which is the equivalence class of the function of n that counts the maximum numbers of (ϵ, n)-separated sets for the holonomy pseudogroup $\mathcal{G}_{\mathcal{F}}$ of $\mathcal{F}$. This invariant is analogous to the leaf growth rate function discussed in the last section. Egashira used this invariant to extend the theory of levels to group actions on the circle of differentiability class C^{1+bv} [135].

For dynamical systems generated by a single diffeomorphism, positive topological entropy implies there is an abundance of hyperbolic behavior in a neighborhood of non-atomic invariant measures. For foliations, the concept of "abundant hyperbolic behavior" also makes sense, and leads to many results and questions. We begin by recalling a series of questions given in Section D of [328] by Christian Bonatti, Rémi Langevin et Claudio Possani.

Given $R > 0$, consider a path γ_x in L_x with origin x and length less or equal to R and project it locally on L_y starting at the point x. Let $p_{\text{loc}}(\gamma)$ be the resulting path on L_y. Perform the same construction with a path starting in y and projecting it onto L_x. Then define

$$d_1 = \sup_{\gamma_x | l(\gamma_x) \leq R} \sup_t (d\gamma_x(t), p_{\text{loc}}\gamma_x(t))$$

$$d_2 = \sup_{\gamma_y | l(\gamma_y) \leq R} \sup_t (d\gamma_y(t), p_{\text{loc}}\gamma_y(t))$$

$$d_R(x, y) = \max(d_1, d_2)$$

The function $R \mapsto d_R(x, y)$ measures how the leaf L_x through a point x goes away from the leaf L_y through a nearby point y.

A foliation $\mathcal{F}$ is **expansive** if there exists $\varepsilon > 0$ such that for each pair of points $x, y \in M$, close enough to allow the above construction, there exists $R > 0$ such that $d_R(x, y) \geq \varepsilon$.

Inaba and Tsuchiya proved in 1992 that codimension 1 expansive foliations of a compact manifold have a resilient leaf [286]. Hurder studied expansive C^1 group actions and foliations in the papers [259, 260], and showed that there is always a resilient leaf with contracting linear holonomy. In higher codimensions, an expansive foliation of class $C^{1+\alpha}$ has uniformly attracting leaves.

The questions of Bonatti, Langevin et Possani used a stronger notion of expansivity.

A foliation $\mathcal{F}$ is *weakly hyperbolic* if there exists $a > 0, \varepsilon > 0, \varepsilon_1 > 0$, such that if the $d(x, y) < \varepsilon_1$ one has :

$$d_R(x, y) \geq \varepsilon \quad \text{for} \quad R \geq a \cdot \text{Log } (\varepsilon / d_0(x, y))$$

where $d_0(x, y)$ is the transverse distance between x and y, (set $R = 0$ in the definition of d_R.

Using the Riemannian metric of M one can also define locally the direction and the strength of the infinitesimal contraction of the leaves, see [38], obtaining a vector field tangent to $\mathcal{F}$.

The foliation is *strongly hyperbolic* if the vector field of infinitesimal contraction has no sink.

PROBLEM 4.4. *If the foliation is weakly hyperbolic for some Riemannian metric on M, is it true that there exists another Riemannian metric such that the foliation is strongly hyperbolic ?*

Recall the definitions of Markov partition from [71, 80, 83, 620].

PROBLEM 4.5. *Does a strongly hyperbolic foliation have a Markov partition ?*

Modifying slightly the definition of expansivity, we can define foliations with "sensitive dependence on initial data", asking that for every x and every neighborhood $v(x)$ of x there exists $y \in v(x)$ and $R(y)$ such that $d_{R(y)}(x, y) \geq \varepsilon$.

PROBLEM 4.6. *Do there exists nice examples of foliations with "sensitive dependence on initial data" which are not hyperbolic ?*

All three of these very interesting questions remain open to the best of the author's knowledge. The short note [24] might have some relevance to the last question.

Given foliated compact manifolds $(M, \mathcal{F})$ and $(M', \mathcal{F}')$, a *restricted orbit equivalence* between $\mathcal{F}$ and $\mathcal{F}'$ is a measurable isomorphism $h \colon M \to M'$ which maps the leaves of $\mathcal{F}$ to the leaves of $\mathcal{F}'$, and the restriction of h to leaves is a coarse isometry for the leaf metrics. Note that h and its inverse are assumed to preserve the Lebesgue measure class, but need not preserve the Riemannian measure. Such a map preserves the Mackey range of the Radon-Nikodyn cocycle [640]. Restricted orbit equivalence also preserves the entropy positive condition, for ergodic $\mathbb{Z}^n$ actions.

PROBLEM 4.7. *Does restricted orbit equivalence preserve geometric entropy? If $h_g(\mathcal{F}) > 0$, must $h(\mathcal{F}') > 0$ also?*

Connes has show that the Godbillon-Vey class, or more precisely the Bott-Thurston 2-cocycle defined by it, can be calculated from the flow of weights for the von neumann algebra $\mathfrak{M}(M, \mathcal{F})$ (see [97], Chapter III.6, [98].) This gives another proof of the theorem of Hurder and Katok [266] that if $GV(\mathcal{F}) \neq 0$ then $\mathfrak{M}(M, \mathcal{F})$ has a factor of type III. The flow of weights is determined by the flow on the Mackey range of the modular cocycle, so that a type III factor corresponds to a ergodic component of the Mackey range with no invariant measure. However, almost nothing else is known about how the flow of weights is related to the topological dynamics of $\mathcal{F}$. In particular, Alberto Candel has asked in [70] whether the existence of a resilient leaf can be proven using properties of the flow of weights.

PROBLEM 4.8. *How is the flow of weights for $\mathcal{F}$ related to the dynamics of $\mathcal{F}$?*

Besides being of interest on its own, exploring the connections between the topological dynamics and the Connes-Krieger flow-of-weights classification of the action will give insights into the study of the dynamics of C^1-group actions.

5. Minimal sets

A minimal set for a foliation $\mathcal{F}$ is a closed saturated subset $\mathbb{K} \subset M$, such that there is no proper closed saturated subset $\mathbb{K}' \subset K$. Equivalently, $\mathbb{K}$ is closed is every leaf of $\mathcal{F}$ in $\mathbb{K}$ is dense in $\mathbb{K}$.

Minimal sets enjoy the strongest type of topological recurrence, and their study was a central focus of dynamical systems of one generator, especially in the works of J. Auslander, L. Auslander, R. Ellis, H. Furstenberg, W.H. Gottschalk, G.A. Hedlund, and M. Morse, to mention a few of the prominent early researchers.

For foliations in general, the study of minimal sets is much more difficult – their study divides into the merely difficult (codimension one) and the impossible (codimension greater than one.) Recall that a minimal set $\mathbb{K}$ is *exceptional* if $\mathbb{K}$ is not a compact leaf, and not an open set.

We recall first a series of venerable questions about minimal sets in codimension one. Let $\mathbb{K}$ be an exceptional minimal set for a codimension one C^2-foliation $\mathcal{F}$ of a compact n-manifold M.

PROBLEM 5.1 (Dippolito)**.** *Let $L \subset \mathbb{K}$ be a semiproper leaf of $\mathcal{F}$, $x \in L$ and let $H_x(L, \mathbb{K})$ be the germinal homology group of L at x relative to $\mathbb{K}$. Prove that $H_x(L, \mathbb{K})$ is infinite cyclic.*

Hector proved in his thesis [226] that the infinite jet of holonomy is infinite cyclic. The more precise form of the problem is to show that $H_x(L, \mathbb{K})$ is generated by a contraction.

PROBLEM 5.2 (Hector)**.** *Prove that $M \backslash \mathbb{K}$ has only finitely many components. That is, show that $\mathbb{K}$ has only a finite number of semi-proper leaves.*

This is known to be false for C^1 foliations.

PROBLEM 5.3. *Show that the Lebesgue measure of $\mathbb{K}$ is zero.*

The measure of $\mathbb{K}$ has is known to be zero for special cases [283, 375, 285].

PROBLEM 5.4. *Show that every leaf of $\mathbb{K}$ has a Cantor set of ends.*

Duminy's Theorem [84] shows that the semiproper leaves of $\mathbb{K}$ must have a Cantor set of ends.

Cantwell and Conlon showed that if $\mathbb{K}$ is Markov (i.e, the holonomy pseudogroup $\Gamma \mid K$ is generated by a (1-sided) subshift of finite type), then all four of the above problems are true [80, 83]. Then they asked

PROBLEM 5.5. *Is every minimal set $\mathbb{K}$ Markov?*

All of these are difficult problems, but have an appeal because they would lead to a complete understanding of the dynamics of codimension one minimal sets.

For higher codimensions, there are very few results. Here are some basic questions:

PROBLEM 5.6. *Let $\mathbb{K}$ be a minimal set for a C^r-foliation of a compact manifold.*

- *Find conditions on $\mathcal{F}$ and $r \geq 0$ which imply that $\mathcal{F}$ admits a unique harmonic measure on $\mathbb{K}$.*
- *When does $\mathcal{F}$ admit a holonomy invariant tranverse measure supported on $\mathbb{K}$?*
- *When does $\mathcal{F}$ admit a unique holonomy invariant tranverse measure supported on $\mathbb{K}$?*

The last two questions have well-known answers for codimension-one foliations, as the existence of a transverse invariant measure gives a holonomy invariant transverse "coordinate" (see Sacksteder [510].) Finding criteria for the uniqueness of harmonic measure on a minimal set is an open question in all codimensions.

6. Tangential LS category

The Lusternik-Schnirelmann category of a topological space X is the least integer k such that X may be covered by k open subsets which are contractible in X. This concept was introduced in the course of research on the calculus of variations in the 1930's [359, 289, 290]. Extensions of LS category have been given for actions of compact groups and for fibrewise spaces (see [89].) In her 1998 thesis [88, 95, 96], Hellen Colman defined two versions of LS category for foliations – the tangential category which measures the topological complexities of leaves, and the transverse category, which is a form of category for the leaf space $M/\mathcal{F}$.

Let $(M, \mathcal{F})$ and $(M', \mathcal{F}')$ be foliated manifolds. A map $f\colon (M, \mathcal{F}) \to (M', \mathcal{F}')$ is said to be foliated if it sends leaves into leaves. A homotopy $H\colon M \times \mathbb{R} \to M'$ is said to be *integrable* if H is a foliated map, considering $M \times \mathbb{R}$ to be foliated with leaves $L \times \mathbb{R}$, $L \in \mathcal{F}$. The notation $\simeq_{\mathcal{F}}$ will denote integrable homotopy. Given an integrable homotopy H, for all $t \in \mathbb{R}$ we have a foliated map $H_t\colon (M, \mathcal{F}) \to (M', \mathcal{F}')$. Moreover, for each $x \in M$ the curve $t \mapsto H_t(x)$ is a leafwise curve in M'. Thus, an integrable homotopy is exactly a homotopy for which all of the "traces" are leafwise curves. As a consequence, if $f \simeq_{\mathcal{F}} g$ then f and g induce the same map between the leaf spaces.

An open subset U of M is tangentially categorical if the inclusion map $(U, \mathcal{F}_U) \hookrightarrow (M, \mathcal{F})$ is integrably homotopic to a foliated map $c\colon U \to M$ which is constant on each leaf of $\mathcal{F}_U$. Here U is regarded as a foliated manifold with the foliation $\mathcal{F}_U$ induced by $\mathcal{F}$ on U. The leaves of $\mathcal{F}_U$ are the connected components of $L \cap U$, where L is a leaf of $\mathcal{F}$.

DEFINITION 6.1. *The tangential category* $\mathrm{cat}_{\mathcal{F}}(M)$ *of a foliated manifold* $(M, \mathcal{F})$ *is the least number of tangentially categorical open sets required to cover* M.

As a foliated coordinate chart is categorical, $\mathrm{cat}_{\mathcal{F}}(M) < \infty$ for M compact. Singhof and Vogt showed in [538] the optimal result:

THEOREM 6.2. *If M is compact, then* $\mathrm{cat}_{\mathcal{F}}(M) \leq \dim \mathcal{F} + 1$.

Moreover, Singhof and Vogt showed that for various classes of foliations, there is also a lower bound $\mathrm{cat}_{\mathcal{F}}(M) \geq \dim \mathcal{F} + 1$, hence equality holds. Colman and Hurder [93] gave further lower bound estimates in terms of cohomology and characteristic classes of $\mathcal{F}$. Hurder gave a generalization in [262] of the Eilenberg and Ganea Theorem [136] that the category of a space $X = K(\pi, 1)$ equals the cohomological dimension of π.

THEOREM 6.3. *Let M be a compact manifold, and assume the holonomy covering of each leaf of $\mathcal{F}$ is contractible, then* $\mathrm{cat}_{\mathcal{F}}(M) = \dim \mathcal{F} + 1$.

In his talk in Kyoto, Elmar Vogt announced a generalization of this result [539]

THEOREM 6.4. *Let M be a compact manifold, and assume the* homotopy *covering of each leaf of $\mathcal{F}$ is contractible, then* $\mathrm{cat}_{\mathcal{F}}(M) = \dim \mathcal{F} + 1$.

There are two kinds of questions about tangential category invariant: how is it related to the geometry of the leaves, and can it be given a more "homotopy theoretic" definition?

PROBLEM 6.5. *Let $\varphi\colon G \times M \to M$ a locally free action on a compact manifold M of a connected Lie group, with real rank $k = rank(G)$. Show that* $\mathrm{cat}_{\mathcal{F}}(M) \geq k$ *for the foliation by the orbits of G.*

There is a subfoliation $\mathcal{F}'$ of $\mathcal{F}$ by the orbits of the maximal $\mathbb{R}$-split torus $\mathbb{R}^k \subset G$. Then $\mathrm{cat}_{\mathcal{F}'}(M) = k$ by [93]. Does this imply $\mathrm{cat}_{\mathcal{F}}(M) \geq k$? (cf. Singhof [536, 537].)

PROBLEM 6.6. *Give a homotopy-theoretic interpretation of* $\mathrm{cat}_{\mathcal{F}}(M)$ *corresponding to the Whitehead and Ganea definitions of category.*

This is one of the most important open problems in the subject. The paper by Colman [92] gives one approach to a solution (for pointed category theory.)

7. Transverse LS category

Let $\mathcal{F}$ be a C^r–foliation of the manifold M, where $r \geq 0$. A saturated subset $X \subset M$ equipped with the restricted foliation $\mathcal{F}_X$ is an example of a *foliated space* (cf. [508, 266, 425, 71].) An open subset $U \subset M$ is regarded as a foliated manifold with the foliation $\mathcal{F}_U$ induced by $\mathcal{F}$. Note that if U is not saturated, then the leaves of $\mathcal{F}_U$ are the connected components of the intersections $L \cap U$, L a leaf in M.

Let $(X, \mathcal{F})$ and $(X', \mathcal{F}')$ be foliated spaces. A homotopy $H: X \times [0,1] \to X'$ is said to be *foliated* if for all $t \in [0,1]$, the map H_t sends each leaf L of $\mathcal{F}$ into another leaf L' of $\mathcal{F}'$.

The concept of foliated homotopy of saturated subsets is connected with many aspects of foliation theory, which the problems in this section will highlight.

An saturated subset $X \subset M$ is *transversely categorical* if there is a foliated homotopy $H: X \times [0,1] \to M$ such that $H_0: X \to M$ is the inclusion, and $H_1: X \to M$ has image in a *single* leaf of $\mathcal{F}$. In other words, the subset X of M is transversely categorical if the inclusion $(X, \mathcal{F}_X) \hookrightarrow (M, \mathcal{F})$ factors through a leaf, up to foliated homotopy.

DEFINITION 7.1. *The transverse saturated category* $\mathrm{cat}_{\pitchfork}(X)$ *of a foliated space* $(X, \mathcal{F})$ *is the least number of transversely categorical, open saturated sets of* M *required to cover* X*. If no such covering exists, then we set* $\mathrm{cat}_{\pitchfork}(X) = \infty$*.*

The basic question is

PROBLEM 7.2. *What foliations have transverse saturated category* $\mathrm{cat}_{\pitchfork}(X) < \infty$*?*

Several results are now known. Colman showed in [88, 95]

THEOREM 7.3. *A compact Hausdorff foliation* $\mathcal{F}$ *of a compact manifold* M *has* $\mathrm{cat}_{\pitchfork}(X) < \infty$*.*

Colman and Hurder [94] gave estimates for the transverse category of compact Hausdorff foliations in terms of the exceptional set of the foliation.

Hurder and Walczak showed in [272]

THEOREM 7.4. *If* $\mathcal{F}$ *is a compact foliation* $\mathcal{F}$ *of a compact manifold* M*, then* $\mathrm{cat}_{\pitchfork}(X) < \infty$ *implies the leaf space* $M/\mathcal{F}$ *is Hausdorff.*

COROLLARY 7.5. *A compact foliation* $\mathcal{F}$ *of a compact manifold* M *has* $\mathrm{cat}_{\pitchfork}(X) < \infty$ *if and only if* $\mathcal{F}$ *is compact Hausdorff.*

There are many examples of foliations with non-compact leaves and finite category. However, in all such cases Hurder showed in [258]

THEOREM 7.6. *A foliation* $\mathcal{F}$ *of a compact manifold* M *with* $\mathrm{cat}_{\pitchfork}(X) < \infty$ *has a compact leaf.*

Thus, the question is whether there are interesting classes of foliations where $\mathrm{cat}_{\pitchfork}(X) < \infty$ corresponds to some geometric property of the foliation. For example, in the case of Riemannian foliations, Colman has given in [91] a criteria for $\mathrm{cat}_{\pitchfork}(X)$ to be infinite, in terms of the geometry of the SRF (Singular Riemannian Foliation) associated to $\mathcal{F}$.

PROBLEM 7.7. *Suppose that* $\mathcal{F}$ *is a Riemannian foliation of a compact manifold* M*. Find geometric conditions on* $\mathcal{F}$ *equivalent to* $\mathrm{cat}_{\pitchfork}(X) < \infty$*.*

Another possible extension is to foliations whose leaves are "tame".

PROBLEM 7.8. *When does a proper foliation* $\mathcal{F}$ *of a compact manifold* M *have* $\mathrm{cat}_{\pitchfork}(X) < \infty$*?*

The Reeb foliation of $\mathbb{S}^3$ is a proper foliation with $\mathrm{cat}_{\pitchfork}(X)$ infinite, while there the (proper) foliation of $\mathbb{T}^2$ with two Reeb components has $\mathrm{cat}_{\pitchfork}(X) < \infty$. The obstruction to covering the Reeb foliation seems subtle.

PROBLEM 7.9. *Does a proper real analytic foliation $\mathcal{F}$ of a compact manifold M have a finite covering with transversely finite category?*

Another way to generalize the definition of category, is to drop the condition that the covering be by open sets. For the category theory of spaces, it is more common to use coverings by categorical closed sets [289, 290]. For foliations, the existence of a covering by either open or closed transversely categorical saturated sets is already a strong hypotheses – the Reeb foliation admits neither.

The Borel algebra $\mathcal{B}(\mathcal{F})$ of $\mathcal{F}$ was introduced in Heitsch and Hurder [241]. This is just the σ–algebra generated by the open saturated sets of M. The point of its introduction in [241] was that the Godbillon measure introduced by Duminy on open sets in codimension one foliations [118, 78, 72] extends to the full σ–algebra $\mathcal{B}(\mathcal{F})$. In fact, Heitsch and Hurder showed that the Godbillon and Weil measures are well-defined on the full measure algebra $\mathcal{M}(\mathcal{F})$.

Define the extended Borel algebra $\widehat{\mathcal{B}}(\mathcal{F})$ of $\mathcal{F}$ to be the σalgebra generated by the sets in $\mathcal{B}(\mathcal{F})$ along with the individual leaves of $\mathcal{F}$. Thus, $\widehat{\mathcal{B}}(\mathcal{F}) \subset \mathcal{M}(\mathcal{F})$.

A *Borel decomposition of* $\mathcal{F}$ is a countable collection of disjoint subset $\{A_n \in \widehat{\mathcal{B}}(\mathcal{F}) \mid n = 1, 2, \dots\}$ so that

$$M = \bigcup A_n$$

For example, the Reeb foliation of $\mathbb{S}^3$ admits a decomposition with three sets, each in $\mathcal{B}(\mathcal{F})$.

A foliation $\mathcal{F}$ is said to admit a *categorical decomposition* if there is a Borel decomposition $\{A_n \in \widehat{\mathcal{B}}(\mathcal{F}) \mid n = 1, 2, \dots\}$ such that each set $A_n \subset M$ is transversely categorical.

While a compact foliation $\mathcal{F}$ has $\mathrm{cat}_{\pitchfork}(X) < \infty$ only if it is Hausdorff, one can use the structure of the Epstein filtration of the exceptional set to show that every compact foliation admits a categorical decomposition.

Tthe Reeb stability for proper leaves in codimesnion one [280] implies that a proper leaf with trivial holonomy has an open categorical neighborhood.

It is natural to ask what other classes of foliations also have such a decomposition. For example, the following result follows immediately from the work of Millett [392]

THEOREM 7.10. *If $\mathcal{F}$ is a proper foliation, then $\mathcal{F}$ admits a categorical decomposition.*

PROBLEM 7.11. *What non-proper foliations $\mathcal{F}$ admit categorical decompositions?*

We mention one more question here, which will receive more explanation in the sections on characteristic classes.

PROBLEM 7.12. *Suppose $\mathcal{F}$ is a C^2-foliation with a categorical decomposition. Show that all of the residual secondary classes of $\mathcal{F}$ vanish.*

8. Foliated Morse theory

"Foliation Theory" and "Morse Theory" are two of the great subjects to blossom in the 1950's, and have since found applications in all areas of mathematics. It is a perennial question to ask how to combine these two. Recent research in LS-category and pseudo-isotopy theory reinforce the interest in this speculative issue. Here is a quote by René Thom from 1964:

> On sait quel puissant moyen de classification des structures différentiables nous est donné par la théorie de Morse, telle qu'elle a été généralisée par des auteurs tels que A. Wallace, S. Smale, etc. .... Il est naturel de penser que cette méthode pourra également se révéler efficace dans l'étude des structures plus fines que sont les variétés feuilletées."
> *René Thom, page 173,* [569]

Morse functions can be used to try to understand the geometry of leaves of the foliation. For example, the 1973 thesis of Steve Ferry [146] studied the existence of open dense sets of function on M which have generic singularities. Ferry and Wasserman [147] showed that a smooth codimension-one foliation on a compact simply connected manifold M has a compact leaf if and only if every Morse function on M has a cusp.

More generally, properties of a foliated Morse function should be related to "topological structures" of the foliation, perhaps in terms of foliated handle structures, or leafwise structures. The tangential and transverse LS categories of a foliation discussed in the previous section provide examples of such topological structures for a foliation.

PROBLEM 8.1. *Give a foliated Morse theory interpretation of the tangential LS category.*

Kazuhiko Fukui gave some new applications of Thom's ideas in [159], where he studied the singularities of foliated Morse functions in codimension greater than two and gave conditions which implies that the foliation must be a fibration. One generalization of this result might be to use Morse Theory to study when a foliation has finite tranverse category, since a local fibration is the model example of a foliation with finite category.

PROBLEM 8.2. *Find conditions on a foliated Morse function which are sufficient to imply that the transverse saturated category of $\mathcal{F}$ is finite.*

Foliated Morse Theory should be related to the classification problem of foliations. An early version of Mather's proof [364, 366] that $\pi_2(B\Gamma_1) = 0$ used Morse functions in the proof (cf. also [336].) The exposition of this proof by Claude Roger [493] makes the role of the Morse function explicit. The Mather-Thurston Theorem greatly generalized these results to all codimensions [364, 365, 366, 367, 368, 369, 571], but the role of Morse functions appears to vanish.

In the early 1980's, Kiyoshi Igusa developed the foundations of parametrized Morse theory in a series of foundational papers [273, 274, 275, 276, 277], several of which consider the problem of properties of Morse functions restricted to leaves of foliations. The work of Igusa is considered a very powerful technical tool in differential topology. The work of Eliashberg and Mishachev on "wrinkled maps" [138, 139, 140] gives a new proof of the existence theorem for foliations on compact manifolds in higher codimensions by Thurston [572].

PROBLEM 8.3. *Can the parametrized Morse theory of Igusa be used to give an alternate proof of the Mather-Thurston theorem?*

The question is vaguely stated, but intuition suggests connections between foliated Morse theory and classification theory of foliations which are waiting to be discovered (cf. [212, 213, 393].)

Alvarez-Lopez developed Morse inequalities for Riemannian foliations in [9], and A. Connes and T. Fack studies foliation Morse theory and leafwise Betti numbers for measured foliations in [99] .

PROBLEM 8.4. *Suppose that $\mathcal{F}$ has a transverse invariant measure μ. Is there a relation between the Morse inequalities for measured foliations and the tangential category $\mathrm{cat}_{\mathcal{F}}(M)$ of $\mathcal{F}$?*

9. Automorphisms of foliated manifolds

The group $\mathrm{Diff}^r(M)$ of C^r-diffeomorphisms of a compact manifold forms a Frechet space, and the works of Joshua Leslie [351, 352, 354, 355, 356] and Hideki Omori [454, 455, 456, 457] showed it has many properties of a Lie group. The survey of Milnor [394] gives an overview of the Lie group aspects, and non-aspects, of $\mathrm{Diff}^r(M)$. The more recent book by Banyaga [25] is an excellent resource, which includes in-depth development of the many topics in the theory of diffeomorphism groups on which he has worked.

The point of this section is to highlight some questions about the structure of the automorphism groups of foliated manifolds, a special area in the above more general subject. This is a venerable research area, though difficult as these groups present numerous technical obstacles to their study.

Let $\mathrm{Diff}^r(M, \mathcal{F})$ denote the C^r-diffeomorphisms of M which send leaves of $\mathcal{F}$ to leaves of $\mathcal{F}$, and $\mathrm{Diff}^r_0(M, \mathcal{F})$ the normal subgroup of $\mathrm{Diff}^r(M, \mathcal{F})$ which are connected to the identity.

We also introduce the group $\mathrm{Diff}^r(\mathcal{F})$ of C^r-diffeomorphisms of M which map each leaf of $\mathcal{F}$ into itself, and the normal subgroup $\mathrm{Diff}^r_0(\mathcal{F})$ of elements path connected to the identity.

Let $\Xi(\mathcal{F})$ denote the space of C^r vector fields tangent to the leaves of $\mathcal{F}$. Then for each $X \in \Xi(\mathcal{F})$ we can form the flow $\exp(X) \in \mathrm{Diff}^r_0(\mathcal{F})$.

These various subgroups of diffeomorphisms are related by inclusions

$$\exp\colon \Xi(\mathcal{F}) \to \mathrm{Diff}^r_0(\mathcal{F}) \subset \mathrm{Diff}^r_0(M, \mathcal{F}) \subset \mathrm{Diff}^r(M, \mathcal{F}) \supset \mathrm{Diff}^r(\mathcal{F})$$

The outer automorphism group of $(M, \mathcal{F})$ is the quotient

$$\mathrm{Aut}^r(M, \mathcal{F}) = \mathrm{Diff}^r(M, \mathcal{F})/\mathrm{Diff}^r_0(\mathcal{F})$$

and contains the subgroup of "leafwise" outer automorphisms

$$\mathrm{Aut}^r(\mathcal{F}) = \mathrm{Diff}^r(\mathcal{F})/\mathrm{Diff}^r_0(\mathcal{F})$$

Leslie studied foliations with a finite number of dense leaves in [353], and proved $\mathrm{Aut}(M, \mathcal{F})$ is finite dimensional in that case.

If the foliation $\mathcal{F}$ on M is obtained from the suspension of a group action $\varphi\colon G \times X \to X$, then the group of centralizers of the action on X injects into $\mathrm{Aut}^r(M, \mathcal{F})$. There are many studies of centralizers of diffeomorphisms (cf. J. Palis and J. C. Yoccoz [460, 461]).

Nathan dos Santos and his colleagues has studied $\mathrm{Diff}^r_0(M, \mathcal{F})$ for foliations defined by actions of abelian groups in a series of papers [12, 13, 14, 360, 116, 117] where they obtained various rigidity and perturbation results.

One of the celebrated applications of Sullivan's theory of minimal models [556, 557, 559] was to show that $\mathrm{Aut}^r(M)$ is a virtually algebraic group. Is it possible to show a similar result for foliations?

PROBLEM 9.1. *Suppose that $\mathcal{F}$ has a finite number of leaves which are dense in M. Show that $\mathrm{Aut}^r(M, \mathcal{F})$ and $\mathrm{Aut}^r(\mathcal{F})$ have algebraic subgroups of finite index.*

The theorems of Witte [630], Ghys [188], and Burger and Monod [65, 66] (cf. also Monod [424]) showed that a C^1 action of a higher rank lattice on the circle must factor through a finite group. Feres and Witte [145] extended this result to show that if a higher rank lattice acts on a foliation which is codimension–one and almost without holonomy, then the action is finite.

PROBLEM 9.2. *Prove that a higher rank lattice which acts on a codimension one foliation must preserve a 1–form on M transverse to $\mathcal{F}$.*

One can pose the more general problem:

PROBLEM 9.3. *Let $\mathcal{F}$ be a codimension one C^r foliation of a compact manifold M. Calculate $\mathrm{Aut}^r(M/\mathcal{F})$ and $\mathrm{Aut}^r(\mathcal{F})$ for $r \geq 1$.*

For codimensions greater than one, there is very little in the literature about $\mathrm{Diff}^r(M, \mathcal{F})$.

PROBLEM 9.4. *Let $\mathcal{F}$ be a C^r foliation of a compact manifold M with codimension $q > 1$. Find geometric conditions on $\mathcal{F}$ which imply $\mathrm{Aut}^r(M/\mathcal{F})$ and $\mathrm{Aut}^r(\mathcal{F})$ are countable groups.*

PROBLEM 9.5. *Let $\mathcal{F}$ be a C^2 foliation of a compact connected manifold M. Suppose that $\mathcal{F}$ has a non-trivial residual secondary class (see §12 following) of the form $\Delta_{\mathcal{F}}(h_I \wedge c_J)$ where $h_I \neq h_1$. Prove that $\mathrm{Aut}^2(M/\mathcal{F})$ is finite.*

The leafwise holonomy of $\mathcal{F}$ must have higher rank by [249].

10. Godbillon-Vey class in codimension one

In 1971, C. Godbillon and J. Vey introduced the invariant $GV(\mathcal{F}) \in H^3(M)$ of a C^2-foliation $\mathcal{F}$ of codimension-one on the manifold M [198], which was then named after them. Previously, one could say the study of foliations was considered either as an area of topology, viewing a foliation is a generalized fibration structure on a manifold (cf. the many geometric results cited in the introduction and in [336]), or as an area within differential equations, whose key results concerned recurrence and limit sets (cf. [509, 510, 511, 490]). With the advent of the Godbillon-Vey class and its generalization to the other secondary classes, a new area of foliation research was created – to understand what these classes "measured", when they were non-vanishing, and their implications for classification theory. Research in this area features the interplay of geometry, topology, topological dynamics, and ergodic theory.

The early investigations (in the 1970's) of the "geometry Godbillon-Vey class" started by selecting a class of codimension-one foliations – without holonomy, or almost without holonomy, or totally proper, etc. – and then proving the Godbillon-Vey class must vanish, and also showing that the foliation is cobordant to zero. We mention the works of K. Fukui [162, 163] G. Hector [226, 227], Mizutani [401], T. Mizutani, S. Morita and T. Tsuboi [404, 405], T. Mizutani and I. Tamura [408], R. Moussu [432], R. Moussu and F. Pelletier [433], R. Moussu and R. Roussarie [434], T. Nishimori [444], F. Sergeraert, [521], T. Tsuboi [578], N. Tsuchiya [599], C. Roger [493], H. Rosenberg and W.P. Thurston [504], and W. Thurston [570].

Moussu and Pelletier [433] and Sullivan [558] made the conjecture that that if the Godbillon-Vey class is non-zero, then the foliation must have a leaf of exponential growth. There was a dichotomy at that time between the examples where the foliation had been proven null-cobordant – these examples had all leaves of polynomial growth – and the foliations where $GV(\mathcal{F}) \neq 0$ – these had an open set of leaves with uniformly exponential growth. The amazing work of Duminy [118, 119] proved, in an amazing leap forward, the vanishing of the Godbillon-Vey class for foliations with only trivial minimal sets (i.e., each minimal set is a compact leaf.) In particular, $GV(\mathcal{F}) \neq 0$ implies there are uncountably many leaves with uniformly exponential growth. There has been no corresponding leap forward to show the null-cobordance of this class of foliations.

On the other hand, Tsuboi studied the cobordism classes of foliations with low differentiability, and showed that every C^1-foliation of codimension one with oriented normal bundle is null-cobordant, an amazing result [578, 580, 582, 583, 584].

The Seminaire Bourbaki "Sur l'invariant de Godbillon-Vey" by Ghys [184], gives an excellent review of developments in the 1980's, and the survey by the author [261] gives a more extensive discussion of the ergodic theory approach to the study of the Godbillon-Vey class and more recent developments.

PROBLEM 10.1. *Prove that if $\mathcal{F}$ is a C^1 foliation of codimension-one on a compact manifold M, and every minimal set of $\mathcal{F}$ is a compact leaf, then $\mathcal{F}$ is null-cobordant.*

There is still open the original question, of just what does the Godbillon-Vey class measure?

PROBLEM 10.2. *Give a* geometric *interpretation of the Godbillon-Vey invariant.*

The Reinhart-Wood formula [492] gave a *pointwise* geometric interpretation of $GV(\mathcal{F})$ for 3-manifolds. What is needed is a more global geometric property of $\mathcal{F}$ which is measured by $GV(\mathcal{F})$. The helical wobble description by Thurston [570] is a first attempt at such a result, and the Reinhart-Wood formula suitably interprets this idea locally. Langevin has suggested that possibly the Godbillon-Vey invariant can be interpreted in the context of integral geometry and conformal invariants [60, 329] as a measure in some suitable sense. The goal for any such an interpretation, is that it should provide *sufficient* conditions for $GV(\mathcal{F}) \neq 0$.

PROBLEM 10.3. *Prove that for a codimension one C^2-foliation $\mathcal{F}$, the Godbillon-Vey measure of an exceptional minimal set is zero.*

This has been proven in "almost all" cases [80, 83, 285]. The problem is to prove it without any extra hypotheses.

Novikov's proof of the topological invariance of the rational Pontrjagin classes of a compact manifold [451, 452, 453] was one of the celebrated theorems of the 1960's. This result was extended to the normal Pontrjagin classes of a foliation by Baum and Connes [30]. It is an outstanding question whether the Godbillon-Vey class has this property too.

The "topological invariance of the Godbillon-Vey class" means that given codimension-one foliations $(M, \mathcal{F})$ and $(M', \mathcal{F}')$, suppose there is a homeomorphism $h\colon M \to M'$ mapping the leaves of foliation $\mathcal{F}$ to the leaves of $\mathcal{F}'$, then $h^*GV(\mathcal{F}') = GV(\mathcal{F})$.

If h is C^1 and the foliations $\mathcal{F}$ and $\mathcal{F}'$ are both C^2, then Raby [484] proved $h^*GV(\mathcal{F}') = GV(\mathcal{F})$. When h and its inverse are both absolutely continuous, then Hurder and Katok [267] proved this.

There are counter-examples to topological invariance of the Godbillon-Vey class if the foliations have differentiability less than C^2. For such foliations, the problem must be phrased using one of the several extensions of the Godbillon-Vey invariant. Ghys defined in [180] a "Godbillon-Vey" invariant for piecewise C^2-foliations in codimension one, and then showed via surgery on Anosov flows on 3-manifolds that there are homeomorphic piecewise C^2-foliations with distinct "Godbillon-Vey" invariants.

Hurder and Katok defined in [267] a "Godbillon-Vey" type invariant for the foliations of fractional differentiability class $C^{1+\alpha}$ where $\alpha > 1/2$. They showed that the weak-stable foliations of volume preserving Anosov flows on 3-manifolds satisfy this condition. Following calculations of Mitsumatsu [397], they showed that for the geodesic flow of a metric of variable negative curvature on a compact Riemann surface, the "Godbillon-Vey" invariant varies continuously and non-trivially as a function of the metric (Corollary 3.12, [267]). The weak-stable foliations of all of these metrics are topologically conjugate.

Tsuboi gave a unified treatment of both of these extensions in [586, 587, 589].

PROBLEM 10.4. *Prove the Godbillon-Vey invariant for C^2 foliations is a topological invariant.*

An intermediate test case might be to assume h and its inverse are a Hölder C^α-continuous for some $\alpha > 0$, and then prove $h^*GV(\mathcal{F}') = GV(\mathcal{F})$, using for example arguments from regularity theory of hyperbolic systems and an approach similar to Ghys and Tsuboi [196].

The concept of "epais" (or "thickness") for codimension-one C^2 foliations was introduced by Duminy [118, 119, 78] and given in terms of the structure theory of C^2-foliations. It was used to show that a foliation with trivial minimal sets admits almost invariant transverse volume forms on an open saturated subset, which is a purely dynamical consideration.

PROBLEM 10.5. *What is meaning of thickness?*

Does the thickness have an interpretation as a dynamical property of the foliation geodesic flow, or some other ergodic property of $\mathcal{F}$?

11. Secondary classes and $B\Gamma_q$

In this section we introduce some basic concepts and results about the secondary classes and the classifying spaces for foliations, which will be assumed in the following sections. An excellent reference for this material is the lecture notes by Lawson [337].

Assume that all foliations and maps between manifolds are at least differentiability class C^2, and $\mathcal{F}$ has codimension q.

Foliations $\mathcal{F}_0$ and $\mathcal{F}_1$ of codimension q on a manifold without boundary M are *integrably homotopic* if there is a foliation $\mathcal{F}$ on $M \times \mathbb{R}$ of codimension q such that $\mathcal{F}$ is everywhere transverse to the slices $M \times \{t\}$, so defines a foliation $\mathcal{F}_t$ there, and $\mathcal{F}_t$ on M_t agrees with $\mathcal{F}_t$ on M for $t = 0, 1$.

If $\mathcal{F}_0$ is integrably homotopic to $\mathcal{F}_1$ then the classifying map for $\mathcal{F}$, given by $\nu\colon M \times \mathbb{R} \to B\Gamma_q$ restricts to a homotopy between the classifying maps $\nu_t\colon M \times \mathbb{R} \to B\Gamma_q$ for $\mathcal{F}_t$, $t = 0, 1$. The Gromov-Phillips theory implies the converse also holds – if M is an open manifold with no compact connected components, then ν_0 homotopic to ν_1 implies $\mathcal{F}_0$ and $\mathcal{F}_1$ are integrably homotopic.

The homotopy fiber $F\Gamma_q$ of the normal bundle map $\nu\colon B\Gamma_q \to BO(q)$ classifies foliations with framed normal bundle. That is, given a foliation $\mathcal{F}$ on M and a framing φ of the normal bundle Q, then we get a map $\nu^{\varphi}\colon M \to F\Gamma_q$ which is well-defined up to homotopy. Note that the space $F\Gamma_q$ is often denoted by $B\overline{\Gamma}_q$ – we prefer the notation $F\Gamma_q$ suggesting the normal bundle is framed.

We say framed foliations $(\mathcal{F}_0, \varphi_0)$ and $(\mathcal{F}_1, \varphi_1)$ are framed integrably homotopic if there is an integrable homotopy $\mathcal{F}$ of $M \times \mathbb{R}$ with a framing φ whose restrictions yield the framings of $\mathcal{F}_0$ and $\mathcal{F}_1$, respectively. The classifying maps $\nu_0^{\varphi}, \nu_1^{\varphi}\colon M \to F\Gamma_q$ are then homotopic. The converse also holds – if M is an open manifold with no compact connected components, then ν_0^{φ} homotopic to ν_1^{φ} implies $\mathcal{F}_0$ and $\mathcal{F}_1$ are framed integrably homotopic.

For a paracompact manifold M without boundary, a continuous map $M \to F\Gamma_q$ determines a framed Haefliger structure on M. A key point of interpretation of this data, stressed in Haefliger [212, 213] and Milnor [393], is that it is completely equivalent to giving a foliation of $M \times \mathbb{R}^q$ which is everywhere transverse to the fibers $M \times \mathbb{R}^q \to M$. Thus, understanding cycles $M \to F\Gamma_q$ is equivalent to understanding the foliations of $M \times \mathbb{R}^q$ transverse to the fibers, up to integrable homotopy. The case of a cycle $M \to B\Gamma_q$ is similar, only now the bundle $E \to M$ is no longer assumed to be a product (framed).

There is a well-known differential graded algebra

$$WO_q = \Lambda(h_1, h_3, \ldots, h_{q'}) \otimes \mathbb{R}[c_1, c_2, \ldots, c_q]_{2q}$$

where the subscript "$2q$" indicates that this is a truncated polynomial algebra, truncated in degrees greater than $2q$, and q' is the greatest off integer $\leq q$. The differential is determined by $d(h_i \otimes 1) = 1 \otimes c_i$ and $d(1 \otimes c_i) = 0$. The monomials

$$h_I \wedge c_J = h_{1_1} \wedge \cdots h_{i_\ell} \wedge c_1^{j_1} \cdots c_q^{j_q}$$

where

$$1_1 < \cdots < i_\ell \ , \ |J| = j_1 + 2j_2 + \cdots + qj_q \leq q \ , \ i_1 + |J| > q \tag{1}$$

are closed, and they span the cohomology $H^*(WO_q)$ in degrees greater than $2q$. The *Vey basis* is a subset of these (cf. [52, 300, 308, 336].)

A foliation $\mathcal{F}$ on M determines a map of differential algebras into the de Rham complex of M, $\Delta_{\mathcal{F}}\colon WO_q \to \Omega^*(M)$. The induced map in cohomology, $\Delta_{\mathcal{F}}^*\colon H^*(WO_q) \to H^*(M)$ depends only the integrable homotopy class of $\mathcal{F}$. The secondary classes of $\mathcal{F}$ are spanned by the images $\Delta_{\mathcal{F}}^*(h_I \wedge c_J)$ for $h_I \wedge c_J$ satisfying (1).

When the normal bundle Q is trivial, the choice of a framing, denoted by φ, enables the definition of additional secondary classes. Define the differential graded algebra

$$W_q = \Lambda(h_1, h_2, \ldots, h_q) \otimes \mathbb{R}[c_1, c_2, \ldots, c_q]_{2q}$$

Again, the monomials

$$h_I \wedge c_J = h_{1_1} \wedge \cdots h_{i_\ell} \wedge c_1^{j_1} \cdots c_q^{j_q}$$

satisfying (1) are closed, and they span the cohomology $H^*(W_q)$ in degrees greater than $2q$.

The data $(\mathcal{F}, \varphi)$ determine a map of differential algebras

$$\Delta_{\mathcal{F}}^{\varphi} \colon W_q \to \Omega^*(M)$$

The induced map in cohomology, $\Delta_{\mathcal{F}}^{\varphi *} \colon H^*(W_q) \to H^*(M)$ depends only on the homotopy class of the framing φ and the framed integrable homotopy class of $\mathcal{F}$.

The constructions above are all "natural" so there exist universal maps

$$\begin{aligned} \Delta^* \colon H^*(WO_q) &\to H^*(B\Gamma_q) \\ \Delta^{\varphi *} \colon H^*(W_q) &\to H^*(F\Gamma_q) \end{aligned}$$

Again, this is described very nicely in Lawson [337].

We also briefly define three types the secondary classes which have been used in the literature: rigid classes, generalized Godbillon-Vey classes, and residual classes.

A foliation $\mathcal{F}$ on M of codimension q extends to a foliation $\mathcal{F}'$ of $M' = M \times \mathbb{R}$ with codimension $q+1$. If $\mathcal{F}$ is framed, then $\mathcal{F}'$ is also framed, where the vector field $\partial/\partial x$ along the factor $\mathbb{R}$ is added to the framing. Correspondingly, there are a natural restriction maps $WO_{q+1} \to WO_q$ and $W_{q+1} \to W_q$ which induce maps on cohomology. The image of $H^*(WO_{q+1}) \to H^*(WO_q)$ is denoted by $\Re O_q$. Similarly, the image of $H^*(W_{q+1}) \to H^*(W_q)$ is denoted by $\Re_q$. The classes in $\Re O_q$ or $\Re_q$ are called *rigid* secondary classes. This is due to a well-known result of Heitsch [234] that the images $\Delta_{\mathcal{F}}(\Re O_q) \subset H^*(M)$ of these classes are invariant under deformation of $\mathcal{F}$.

More is true – it is immediate that

PROPOSITION 11.1. *Let $z \in \Re_q$, then $\Delta_{\mathcal{F}}^{\varphi *}(z)$ is a framed foliated homotopy invariant.*

As discussed in [246, 248], there are examples of foliations with framed normal bundles in all even codimensions at least 4 which have non-vanishing rigid classes.

A class $h_I \wedge c_J$ is said to be *residual* if the degree of the Chern component c_J is $2q$. That is, if $|J| = q$. The key property of a residual class is that the form $\Delta_{\mathcal{F}}^{\varphi}(z) \in \Omega^*(M)$ has maximal "transverse weight" q, so has properties of a generalized measure on the measure algebra $\mathcal{M}(\mathcal{F})$ introduced in §7. This is discussed further in [118, 241, 251, 266] and also in volume II of Candel and Conlon [72]. Residual classes could also be called "residuable" classes - except that the word is effectively unpronounceable – because it is these classes for which the residue approach applies for calculating the secondary classes. The residue method dates back to Grothendieck; its application to foliations began with vector field residue theorems of Bott [39, 40], Baum and Cheeger [29], and Baum and Bott [28]. Heitsch developed a residue theory for smooth foliations [236, 237, 238, 239] which was essential to his calculations of the non-vanishing of the secondary classes. Residue theory was further extended by Lehmann to singular foliations and Intersection Cohomology (cf. [350] for a survey of his work.)

Finally, a class $h_I \wedge c_J$ with $h_I = h_1$ – that is, the index set I is the singleton $\{1\}$ – is called a *generalized Godbillon-Vey class*. The conditions (1) imply that the degree of the Chern component c_J is $2q$, so these classes are a subset of the residual classes. One example is the class $h_1 \wedge c_1^q$ which is the standard extension of the Godbillon-Vey class to codimension q.

12. Geometry and dynamics of the secondary classes

The first question asked about the secondary classes thirty years ago, is the first problem in this section, as it is still a good question.

PROBLEM 12.1. *Show there exists a C^2-foliation $\mathcal{F}$ such that $\Delta^*_{\mathcal{F}}(z) \in H^*(M)$ is non-zero for every non-zero $z \in H^*(WO_q)$.*

PROBLEM 12.2. *Show there exists a C^2-foliation $\mathcal{F}$ with framed normal bundle such that $\Delta^{\varphi *}_{\mathcal{F}}(z) \in H^*(M)$ is non-zero for every non-zero $z \in H^*(W_q)$.*

There were many constructions of examples that realized the secondary classes, by Baker [22], Heitsch [237, 240], Kamber and Tondeur [307, 311, 314], Lazarov [338], Morita [426], Pittie [473, 474] Rasmussen [485, 486], Tabachnikov [562, 563], and Yamato [636, 637].

There is also a long tradition of showing that the Godbillon-Vey class must vanish for certain classes of foliations, and several authors have proven vanishing theorems for this to the higher secondary classes. Pittie showed in [474] that all of the secondary classes vanish if the foliation is defined by a locally homogeneous action of a connected nilpotent Lie group H, and the rigid secondary classes vanish if the group H is solvable. This vanishing was part of the motivation for vanishing theorems in Hurder and Katok [266].

The category theory discussed above suggests a new form of vanishing result. Examples and other calculations suggest that the following:

PROBLEM 12.3. *Let $\mathcal{F}$ be a C^2-foliation on a smooth manifold M. Suppose that the transverse saturated category $\mathrm{cat}_{\pitchfork}(M, \mathcal{F})$ is finite. Prove that all secondary classes must vanish for $\mathcal{F}$. That is, show that $\Delta_{\mathcal{F}} \colon H^*(WO_q) \to H^*(M)$ is the trivial map.*

If the normal bundle has a framing φ, prove that $\Delta^{\varphi *}_{\mathcal{F}} \colon H^*(W_q) \to H^*(M)$ is the trivial map.

Such a result would offer a new approach to vanishing theorems for the secondary classes, complementary to the standard methods and results (cf. [251]) generalizing the Moussu–Pelletier [433] and Sullivan Conjecture [558]. In fact, vanishing theorems for the secondary classes based on categorical open sets is suggestive of the SRH approach to the Godbillon-Vey class developed by Nishimori [444] and Tsuchiya [598, 599]. The idea should be that categorical open sets decompose the foliation into categorical "blocks" which can then be analyzed – exactly analogous to the SRH approach to Godbillon-Vey.

PROBLEM 12.4. *Show that the image of the rigid secondary classes vanish in $H^*(B\Gamma_q)$. That is, show that the composition*

$$H^*(WO_{q+1}) \longrightarrow H^*(WO_q) \xrightarrow{\Delta} H^*(B\Gamma_q)$$

is the trivial map.

Another way to ask the question, is whether every foliation of codimension q on an open manifold is homotopic to a foliation with trivial Haefliger structure. The known non-zero rigid secondary classes are all invariants of the framed homotopy class, but not defined for the general case.

The Weil measures [241] provide a mechanism for localizing a *residual secondary class*, one for which the degree of the term c_J equals the maximal non-vanishing degree 2q, to an $\mathcal{F}$-saturated Borel subset of M. The terminology was chosen because these are also the classes for which there are residue formulas used to calculate them in examples [236, 238, 239]. The residues capture essential normal information about the foliation, so it would be interesting to develop a residue theory for sets more general than submanifolds. This has been done in part by Lehmann [350], but mostly for semi-algebraic sets.

PROBLEM 12.5. *Derive a (measurable) residue theory for the localization of the secondary classes to saturated Borel subsets in $\mathcal{B}(M)$, and relate the formulas to the dynamics of $\mathcal{F}$.*

In particular, it is likely that there are vanishing theorems for the residues which reflect the geometry of the invariant set $X \in \mathcal{B}(M)$. Such vanishing would generalize the several results known for the secondary classes of subfoliations (or multifoliations) (cf. [85, 100, 101, 109, 541, 561, 631]).

Domínguez has developed a residue theory for subfoliations in [115]. It is then natural to propose:

PROBLEM 12.6. *Develop a theory of Weil measures for subfoliations.*

A final question is about the dynamical meaning of the higher secondary classes. Hurder [249] showed that for a C^2-foliation of codimension $q > 1$, if there is a leaf L whose linear holonomy map $D\varphi\colon \pi_1(L, x) \to \mathrm{GL}(\mathbb{R}^q)$ has non-amenable image, then $\mathcal{F}$ has leaves of exponential growth. The proof actually constructs a modified ping-pong game for $\mathcal{F}$, using the C^2-hypothesis to show that the orbits of the holonomy pseudogroup shadow the orbits of the linear holonomy group which has an actual ping-pong game by Tits [574]. Thus, it seems probable that this proof also shows $h_g(\mathcal{F}) > 0$ with these hypotheses. Since the Weil measures vanish for a foliation whose transverse derivative cocycle $D\varphi\colon \Gamma \to \mathrm{GL}(\mathbb{R}^q)$ has amenable algebraic hull [266, 548, 549, 645], it may be possible to combine the methods of [249, 266, 259] to show

PROBLEM 12.7. *If $\mathcal{F}$ is a C^2 foliation of codimension $q > 1$, and there is some non-zero secondary class (or possibly Weil measure), prove that $\mathcal{F}$ has positive geometric entropy, $h_g(\mathcal{F}) > 0$.*

13. Generalized Godbillon-Vey classes

Recall that in higher codimensions $q > 1$, the space of secondary characteristic classes $H^*(W0_q)$ is spanned by monomials of the form $h_I \wedge c_J$ where where $h_I = h_{i_1} \wedge \cdots \wedge h_{i_\ell}$ for $I = (i_1 < ... < y_\ell)$ with each $1 \leq i_k \leq q$ an odd integer. The generalized Godbillon-Vey classes are those of the form $h_1 \wedge c_J$ where c_J has degree $2q$.

A foliation $\mathcal{F}$ is *amenable* if the Lebesgue measurable equivalence relation it defines on $V \times V$ is amenable in the sense of Zimmer [639]. Hurder and Katok showed in [266] that if $y_I \wedge c_J$ is *not* a generalized Godbillon-Vey class, then $\Delta(y_I \wedge c_J) = 0$ for an amenable foliation. The Roussarie examples [198], given by the weak-stable foliation of the geodesic flow for a metric of constant negative curvature, are amenable and have $\Delta(y_1 \wedge c_J) \neq 0$, so this result does not extend to all of the classes. Here are two open questions about the generalized Godbillon-Vey classes.

PROBLEM 13.1. *Suppose that Γ is a finitely generated amenable group and $\alpha : \Gamma \to$ Diff$^{(2)}(N)$is a C^2-action on a compact manifold N without boundary. Let M be a compact manifold whose fundamental group maps onto Γ, $\pi(M, x_0) \to \Gamma$, with associated normal covering $\widetilde{M}$. Form the suspension foliation $\mathcal{F}_\alpha$ on the manifold*

$$V = M_\alpha = (\tilde{M} \times N)/\{(\gamma x, y) \sim (x, \alpha(\gamma)y) \ \ for \ \ \gamma \in \Gamma\}$$

Show that each generalized Godbillon-Vey class $\Delta(h_1 \wedge c_J) \in H^{2q+1}(V; \mathbf{R})$ must vanish for $\mathcal{F}_\alpha$.

If the foliation $\mathcal{F}_\alpha$ admits a homology invariant transverse measure which is *good* (i.e., positive on open transversals) and absolutely continuous, then $\Delta(h_1 \wedge c_J) = 0$ by results of either [241] or [266]. The hypothesis that Γ is amenable implies that there is a good invariant measure for the action on N, and hence also for the suspension foliation $\mathcal{F}_\alpha$. The point of the problem is to prove the vanishing for good measures without the assumption that it is also absolutely continuous. This problem ought not be hard, but some new technique for "regularizing" invariant measures for foliations is required. Section 4 of [241] has a further discussion of this issue.

In §3 above we defined the notions of sub-exponential growth, non-exponential growth and exponential growth for a leaf.

PROBLEM 13.2. *Show that if almost every leaf of $\mathcal{F}$ has non-exponential growth, then every generalized Godbillon-Vey class $\Delta(h_1 \wedge c_J) \in H^{2q+1}(M)$ vanishes for $\mathcal{F}$.*

The conclusion $\Delta(h_1 \wedge c_J) = 0$ was proven in [251] when almost every leaf has sub-exponential growth. For codimension one, Duminy's results show that that $\Delta(y_1 \wedge c_1) \neq 0$ implies there exists an exceptional minimal set of positive measure, and hence the set of leaves with exponential growth also has positive measure. Thus, the question is really for codimension greater than one.

14. Homotopy theory of $B\Gamma_q$

Let $B\Gamma_q$ (respectively, $B\Gamma_q^+$) denote the Haefliger classifying space of C^2, codimension-q foliations (respectively, with orientable normal bundle.) Let $F\Gamma_q$ denote the homotopy fiber of the classifying map of the normal bundle $\nu\colon B\Gamma_q^+ \to B\mathbf{SO}(q)$.

PROBLEM 14.1. *Determine the homotopy type of $F\Gamma_q$, and the structure of the fibration*

$$F\Gamma_q \to B\Gamma_q^+ \to B\mathbf{SO}(q)$$

Almost by definition, $F\Gamma_q$ is $(q-1)$-connected [212, 213]. Haefliger showed more, that $F\Gamma_q$ is q-connected, by constructing an explicit integrable homotopy from any framed Haefliger structure on $\mathbb{S}^q$ to the trivial Haefliger structure. Much more subtle is that $F\Gamma_q$ is $(q+1)$-connected, which follows from the Mather-Thurston Theorem [364, 368, 369, 571] and the simplicity of $\mathrm{Diff}^2_c(\mathbb{R}^q)$.

PROBLEM 14.2. *Prove that $F\Gamma_q$ is $2q$-connected.*

Towards this goal, there is a more specific problem, whose solution would be extremely important for understanding the homotopy theory of $B\Gamma_q$. Let $\iota\colon \mathbf{SO}(q) \to F\Gamma_q$ denote the inclusion of the fibre over the base point of $B\Gamma_q^+$.

The following question was asked by Vogt in the Rio problem session (Problem F.2.1)

PROBLEM 14.3 (Vogt)**.** *Is the map ι homotopic to a constant?*

The map ι defines a framed Haefliger structure over $\mathbf{SO}(q)$ which has an explicit description. Define the foliation $\mathcal{F}_1$ on the product manifold $M = \mathbf{SO}(q) \times \mathbb{R}^q$ with leaves $L_1(\vec{x}) = \{(A, A\vec{x}) \mid A \in \mathbf{SO}(q)\}$ for $\vec{x} \in \mathbb{R}^q$. The standard framing of $\mathbb{R}^q$ defines a normal framing of $\mathcal{F}_1$. In this foliation of the trivial (product) bundle $\mathbf{SO}(q) \times \mathbb{R}^q \to \mathbf{SO}(q)$ the framing is constant, while the leaves of $\mathcal{F}$ twist around the origin. Note that all leaves of $\mathcal{F}$ are compact, and homotopic to the zero section.

Define $\mathcal{F}_0$ to be the product foliation on $\mathbf{SO}(q) \times \mathbb{R}^q$ with leaves $L_0(\vec{x}) = \mathbf{SO}(q) \times \{\vec{x}\}$ for $\vec{x} \in \mathbb{R}^q$, and give it the trivial framing also. Note that there is a diffeomorphism of M which carries the leaves of $\mathcal{F}_0$ to the leaves of $\mathcal{F}_1$, but the diffeomorphism twist the framing.

PROBLEM 14.4. *Construct an explicit framed integrable homotopy from $\mathcal{F}_1$ to $\mathcal{F}_0$. That is, give a C^2, codimension-q framed foliation $\mathcal{F}$ on $M \times [0,1]$ which is everywhere transverse to the slices of $M \times \{t\}$, and restricts to $\mathcal{F}_t$ on $M \times \{t\}$ for $t = 0, 1$.*

The existence of such a foliation is known (abstractly) for $q \leq 4$. Vogt remarked that the case $q = 2$ is relatively easy – it is a good exercise.

A general solution for $n > 2$ is equivalent to proving the existence of a lifting $\tilde{g}$ of the adjoint of the natural map $\mathbf{SO}(q) \to \Omega B\mathbf{SO}(q)$ in the diagram:

$$\begin{tikzcd} & B\Gamma_q^+ \arrow[d, "\nu"] \\ \Sigma\mathbf{SO}(\mathbf{q}) \arrow[ur, "\tilde{g}"] \arrow[r, "g"] & B\mathbf{SO}(\mathbf{q}) \end{tikzcd}$$

One corollary of a solution to this problem would be a solution to the following well-known problem:

PROBLEM 14.5. *Show that $H^{2m}(B\mathbf{SO}(q);\mathbb{R}) \to H^{2m}(B\Gamma_q^+;\mathbb{R})$ is injective for $2m \leq 2q$.*

This is deduced using that the normal bundle map $B\Gamma_q^+ \to B\mathbf{SO}(\mathbf{q})$ is natural with respect to products, the example of Morita [427], and product formulas in cohomology.

Consider the Puppe sequence of homotopy fibrations

$$\cdots \to \Omega F\Gamma_q \to \Omega B\Gamma_q^+ \to \mathbf{SO}(q) \simeq \Omega B\mathbf{SO}(\mathbf{q}) \to F\Gamma_q \to B\Gamma_q^+ \to B\mathbf{SO}_q \tag{2}$$

The following conjecture is motivated by results of [256], where it is proven for $q \leq 4$.

PROBLEM 14.6 (Conjecture 3.4, [256])**.** *Prove that* $\Omega B\Gamma_q^+ \simeq \mathbf{SO}(q) \times \Omega F\Gamma_q$ *for all* $q \geq 1$.

The above questions, and this conjecture, suggests that $\Omega B\Gamma_q^+$ is primal. An explicit model for the space of loops was given by Jekel [293], though it is not clear how to apply his constructions for the above problem.

PROBLEM 14.7 (Vogt)**.** *Let* $\mathfrak{k}$ *be a field of characteristic* 0*, and let* $\iota\colon \mathbf{SO}(q) \to F\Gamma_q$ *be the map described above. Is the induced map in reduced homology with coefficients in* $\mathfrak{k}$ *the zero map ?*

Vogt pointed out that for $\mathfrak{k} = \mathbb{Z}/p\mathbb{Z}$ the answer to the question is "yes" since

$$H^*(B\mathbf{S0}(q); \mathbb{Z}/p\mathbb{Z}) \to H^*(B\Gamma_n^+; \mathbb{Z}/p\mathbb{Z})$$

is injective by a theorem of Bott and Heitsch [53].

We continue with the comments by Vogt from Rio [328]. A theorem of M. Unsöld [607] asserts that a positive answer to this question for a field $\mathfrak{k}$ with characteristic $\neq 2$ will imply that

$$H_*(\Omega F\Gamma_q; \mathfrak{k}) \to H_*(\Omega B\Gamma_q^+; \mathfrak{k}) \to H_*(\mathbf{SO}(q); \mathfrak{k})$$

is a short exact sequence of Hopf algebras. In particular, taking $\mathfrak{k} = \mathbb{Q}$, in the category of topological spaces

$$\Omega B\Gamma_q \simeq \mathbf{SO}(q) \times F\Gamma_q$$

would be true rationally (but this cannot happen as Hopf spaces).

Since the map induced by ι in homology is multiplicative, it suffices to check that ι_* vanishes on generators of the Hopf algebra $H_*(\mathbf{SO}(q); \mathfrak{k})$. Thus again for dimensional reasons the answer to the last question is "yes" for $n = 6$. But the smallest dimension where the answer is unknown is $n = 5$. Here one would have to answer the following

PROBLEM 14.8. *Does the codimension 5 framed Haefliger structure induced on* $\mathbb{S}^7$ *from the above mentioned framed Haefliger structure on* $\mathbf{SO}(5)$ *by the generator of* $\pi_7(\mathbf{SO}(5))$ *bound homologically as a framed Haefliger structure?*

Because of the known connectivity of $F\Gamma_5$ this is equivalent, up to torsion not recognized by $\mathfrak{k}$, to showing that this framed Haefliger structure on $\mathbb{S}^7$ is trivial. This in turn implies directly that at least rationally the map induced by ι in homotopy is surjective for $n = 5$. These comments are continued in the next section on the non-vanishing of normal Pontrjagin classes.

Thurston showed that $GV\colon \pi_3(B\Gamma_1^+) \to \mathbb{R}$ is surjective, which implies that the homotopy theory of $F\Gamma_1 = B\Gamma_1^+$ is a very massive, and possibly intricately constructed topological space. One possibility is that $F\Gamma_1$ has the homotopy type of an Eilenberg-MacLane space $K(\mathbb{R}, 3)$. In this case, however, there should be uncountably many new secondary classes obtained from products of the generators of the integral cohomology $H^3(K(\mathbb{R}, 3); \mathbb{Z})$. These are the discontinuous invariants of Morita [428], and it is unknown if they are non-trivial.

Takashi Tsuboi asked in Question F.4.1 in Rio [328] whether the first of these classes are non-trivial:

PROBLEM 14.9. *Construct a* C^2 *codimension 1 foliation* $\mathcal{F}$ *of* $\mathbb{S}^3 \times \mathbb{S}^3$ *such that its Godbillon Vey invariant* $GV(\mathcal{F}) = (a, b) \in H^3(S^3 \times S^3)$ *with* a/b *being irrational.*

Tsuboi subsequently gave a partial answer in [592] – he showed that the extended Godbillon-Vey class can take on irrational values. As pointed out by Robert Wolak in his AMS review of this article, I.M. Gel'fand, B.L. Feĭ gin, and D.B. Fuks, [174] ask whether the ratio a/b must be rational. This problem remains open.

If $K(\mathbb{R},3)$ is not an Eilenberg-MacLane space, then some of these products may vanish, and then there are enormous families of Whitehead products in $\pi_*(K(\mathbb{R},3))$, or what is equivalent, products in the homology of the H-space $\Omega K(\mathbb{R},3)$. It is completely unknown if such products are non-trivial – though it is interesting to compare it to the ideas in the thesis of Herb Shulman [530, 532].

For higher codimensions, the homotopy groups $\pi_n(B\Gamma_q^+)$ map onto $\mathbb{R}^N$ for infinitely many values of n, and $N \to \infty$ as $n \to \infty$ [218, 245, 248]. In this case, there are non-trivial Whitehead products corresponding to the product of linearly independent secondary classes (an option that does not exists for codimension one, where there is only the Godbillon-vey class). Thee are also *discontinuous invariants* in the sense of Morita [428, 429], and nothing is known about whether they are non-zero.

The Mather-Thurston Theorem gives information about the homotopy theory of $F\Gamma_q$ via the isomorphism $H^*(\Omega^q F\Gamma_q) \cong H^*(B\overline{\mathrm{Diff}_c^2}(\mathbb{R}^q))$. This isomorphism is used to prove that $F\Gamma_q$ is $(q+1)$-connected, but it also has other implications for the homotopy type of $F\Gamma_q$. One of these is to suggest the analogy between its homotopy groups, and the cohomology of Lie groups made discrete $H^*(B\overline{G};\mathbb{Z})$, where $\overline{G}$ is the homotopy fiber of the inclusion $G^\delta \to G$. (For more discussion on this topic, see [64, 220, 369, 394] for example.)

The cohomology groups of Lie groups $H^*(B\overline{G};\mathbb{Z})$, are closely connected to many deep aspects of mathematics, from number theory to geometry [124]. Moreover, they have been computed in many case, in contrast to what has been done for foliations. J. Dupont has also considered discontinuous invariants in the group cohomology [123].

Peter Boullay showed in his thesis [57] that $\pi_5(F\Gamma_2)$ contains a divisible subgroup which maps onto $\mathbb{R}^2$ via the two generalized Godbillon-Vey classes. This is a remarkable result – the proof uses the scissors congruence results of J. L. Dupont, W. R. Parry and C.-H. Sah [128] to compute various relative homology groups for geometrically constructed subgroups of $\mathrm{Diff}^+(\mathbb{S}^2)$. This work suggests many questions.

PROBLEM 14.10. *Is $\pi_n(F\Gamma_q)$ divisible for all $n > 0$?*

Boullay's work also suggests trying to extend it to all codimensions:

PROBLEM 14.11. *Show that $\pi_{2q+1}(F\Gamma_q)$ contains a divisible subgroup which maps onto $\mathbb{R}^N$.*

We conclude these questions about $F\Gamma_q$ with two problems which are variants on problems 14.4 and 14.6.

PROBLEM 14.12. *Let $\mathcal{F}$ be a foliation of an open manifold M defined by a fibration $M \to B$, and φ a framing of the normal bundle to $\mathcal{F}$ on M. Show that the classifying map $\nu^\varphi \colon M \to F\Gamma_q$ is homotopic to zero.*

The action of the loop space $\Omega B\mathbf{SO}(q) \cong \mathbf{SO}(q)$ on the homotopy fiber $F\Gamma_q$ of the map $B\Gamma_q^+ \to B\mathbf{SO}(q)$ in the Puppe sequence (2) can be identified with the universal gauge action on framed bundles (cf. [246, 248, 256]). This gauge action can be non-trivial on cycles with non-trivial secondary classes. However, if the underlying foliation is trivial, it is not known if the action is trivial.

PROBLEM 14.13. *Let $\mathcal{F}$ be a foliation of an open manifold M defined by a fibration $M \to B$, where B has trivial tangent bundle, and φ a framing of the normal bundle to $\mathcal{F}$ on M. Show that the classifying map $\nu^\varphi \colon M \to B\overline{\Gamma}_q$ is homotopic to zero.*

Of course, this is simply a restatement of Problem 14.6, but with this formulation, it seems tractable.

15. Transverse Pontrjagin Classes

Assume that $\mathcal{F}$ is a C^2 foliation of codimension q, with oriented normal bundle Q. The Bott Vanishing Theorem [41, 42] implies that any Pontrjagin class $p_J(Q) = p_1(Q)^{j_1} \cdots p_q^{j_q}(Q)$ of degree $4|J| = 4(j_1 + 2j_2 + \cdots + qj_q) > 2q$ must vanish. For even codimension $q = 2m$, the powers of the Euler class of the normal bundle, $\chi(Q)^k \in H^{kq}(M;\mathbb{R})$, must vanish if $k \geq 4$, as $\chi(Q)^4 = p_q(Q)^2 = 0$.

The map $\nu^* \colon H^\ell(B\mathrm{SO}(q);\mathbb{R}) \to H^\ell(B\Gamma_q^+;\mathbb{R})$ is injective for $\ell \leq q+2$ as the homotopy fiber $F\Gamma_q$ is $(q+1)$-connected. In particular, the universal class $p_1 \in H^4(B\Gamma_2^+;\mathbb{R})$ is non-zero. Morita observed in [426] that there is a compact complex surface M with 2-dimensional sub bundle $Q \subset TM$ such that $p_1(Q) \neq 0$. The 3-connectivity of $F\Gamma_2$ implies that Q is normal to a haefliger structure on M, so by the Thurston Realization Theorem [572], there is a foliation $\mathcal{F}$ on M with normal bundle homotopic to Q.

PROBLEM 15.1. *Give an explicit construction of a C^2 foliation $\mathcal{F}$ of codimension 2 on a compact manifold M such that $p_1(Q) \in H^4(M;\mathbb{R})$ is non-zero.*

The use of Thurston's theorem above is "non-constructive", as it ultimately appeals to the simplicity of $\mathrm{Diff}_c^2(\mathbb{R}^2)$ for the proof. The question is whether there is a direct geometric construction, say starting with a foliation in a normal neighborhood of the 2-skeleton of the manifold used by Morita, which is then extended to a foliation by some explicit geometric method. This is essentially the lowest dimension and codimension where a constructive version of the Thurston's theorem can be sought.

Morita also remarked in [426] that the Cartan formula for the primary classes and the Splitting Principle [395] imply that there are many non-vanishing Pontragin classes under the composition

$$H^\ell(B\mathrm{SO}(2m);\mathbb{R}) \to H^\ell(B\Gamma_{2m}^+;\mathbb{R}) \to H^\ell(B\Gamma_2^+ \times \cdots \times B\Gamma_2^+;\mathbb{R})$$

However, the splitting principle is not sufficient to prove the map ν^* is injective, as can be seen in the simplest case in codimension $q = 4$. By the above, neither p_1^2 nor p_2 get mapped to zero, but by the product formula, $\nu^*(p_1^2 - 2p_2)$ is zero in $H^8(B\Gamma_2^+ \times B\Gamma_2^+;\mathbb{R})$. Vogt posed the following problem in the Rio meeting.

PROBLEM 15.2. *Show there exist a codimension 4 transversely oriented Haefliger structure such that $\nu^*(p_1^2 - 2p_2)$ is not trivial rationally.*

Note that a solution to Problem 14.4 will provide a solution to Problem 15.2.

Ken Millet posed the following problem about the normal Pontrjagin classes in [391]:

PROBLEM 15.3. *Let $\mathcal{F}$ be a compact foliation of a compact manifold. Show that $p_j(Q) = 0$ if the degree is greater than the codimension. That is, $4|J| > q \Longrightarrow p_j(Q) = 0$.*

A compact Hausdorff foliation is Riemannian, so the strong vanishing theorem of Joel Pasternack [465] implies this is true for compact Hausdorff foliations.

Assume that $\mathcal{F}$ has codimension $q = 2m$. We include one question about the cube of the Euler class, as it fits with the other questions about realizing the non-triviality of the Pontrjagin classes.

PROBLEM 15.4. *Construct a C^2 foliation $\mathcal{F}$ with $\chi(\mathcal{Q})^3 \neq 0$.*

The existence of such a foliation is implied by the conjecture that $F\Gamma_q$ is $2q$-connected.

16. Transverse Euler Class

Assume that $\mathcal{F}$ is a C^2 foliation of codimension $q = 2m$, with oriented normal bundle Q. The transverse Euler class $\chi(Q) \in H^q(M;\mathbb{R})$ has a nature intermediate between a primary (Pontrjagin type) class, and a secondary class. For example, the non-vanishing of the Euler class of the normal bundle, $\chi(Q) \in H^q(M;\mathbb{R})$, is often related to the dynamics of $\mathcal{F}$.

Assume that the tangent foliation $T\mathcal{F}$ is also oriented. Let μ be a holonomy-invariant transverse measure for $\mathcal{F}$, and $[C_\mu] \in H_p(V;\mathbb{R})$ the Ruelle-Sullivan class [508] associated to μ.

The *average transverse Euler class* is the pairing $\chi(Q) \cap [C_\mu] \in H_{p-q}(V;\mathbb{R})$. This was introduced by Mitsumatsu [396], analogous with the average Euler class of leaves introduced by Phillips and Sullivan [471], when μ is defined by an averaging sequence in some leaf L.

Recall that a "foliated bundle" is a smooth fibration $\pi\colon M \to X$ with a foliation $\mathcal{F}$ on M that is everywhere transverse to the fibers of π. Hirsch and Thurston proved in [244] that the Euler class of a foliated bundle with compact fiber vanishes if there is a transverse invariant measure. The hypothesis that he fiber is compact is clearly necessary. A flat vector bundle $E \to X$ has a foliation $\mathcal{F}$ transverse to the fibers, and the normal of $\mathcal{F}$ along the zero section $X \to E$ is canonically identified with E. In particular, we can choose E with non-zero Euler class. Let μ be the transverse invariant measure associated with the zero section, then also $\chi(Q) \cap [C_\mu] \neq 0$.

We say that an invariant transverse measure μ for $\mathcal{F}$ is *without atoms* if μ is zero on compact leaves.

In general, an invariant measure without atoms is "diffused" around each of its points, so intuitively, the action of the leafwise germinal holonomy on its normal disc bundle is analogous to a foliated bundle with invariant measure. Continuing the analogy., the Hirsch Thurston theorem suggest that the Euler class should then vanish.

PROBLEM 16.1. *Show that the* average transverse Euler class $\chi(Q) \cap [C_\mu] \in H_{p-q}(V;\mathbb{R})$ *vanishes whenever μ is without atoms.*

Special cases of the problem have been established in [270, 271]. The general case of the problem has proven quite hard to solve, in spite of repeated attempts. Close examination reveals it involves very interesting questions about foliation geometry.

The result implies that if $\mathcal{F}$ has no compact leaves then $\chi(Q) \cap [C_\mu] = 0$. Also, if the measure μ is a smooth; i.e., is a absolutely continuous with respect to Lebesgue measure on M, then this implies then $\chi(Q) \cap [C_\mu] = 0$. As an example, this then implies the theorem of Sullivan [554], that the Euler class vanishes for an $\mathbb{R}^q$ bundle with holonomy in $SL(\mathbb{R}^q)$.

Deligne and Sullivan proved in [106] that a complex vector bundle with discrete structure group over a compact polyhedron is virtually trivial.

PROBLEM 16.2. *Let $\mathcal{F}$ be a holomorphic foliation of a compact manifold M. If $\mathcal{F}$ has a dense leaf, prove that all of the rational Chern classes of the normal bundle vanish.*

The normal bundle to the leaves of $\mathcal{F}$ are complex vector bundles with discrete structure groups, while the dense leaf hypothesis often works as a substitute for compactness.

17. Books on Foliations

The subject of foliations has a plethora of texts which provide introductions to all facets of research in the field. We collect them here just to call attention to the various texts and monographs.

- ♢ **Complex manifolds without potential theory**, [86]
  by Shing-Shin Chern (1969) Second Edition (1979)
- ♢ **Characteristic classes**, [395]
  by John Milnor and James Stasheff (1974)
- ♢ **The Quantitative Theory of Foliations**, [337]
  by H. Blaine Lawson (1975)
- ♢ **Foliated bundles and characteristic classes**, [308]
  by Franz Kamber and Philippe Tondeur (1975)
- ♢ **Gel'fand-Fuks Cohomology and Foliations**, [48]
  by Raoul Bott (Notes by Mark Mostow and Herb Shulman) (1975)
- ♢ **Characteristic classes of foliations**, [473]
  by Harsh Pittie (1976)
- ♢ **Topology of foliations: an introduction**, [568]
  by Itiro Tamura (1976) (transl. 1992)
- ♢ **Geometric Theory of Foliations**, [69]
  by Caesar Camacho and Alcides Lins Neto (1979) (transl. 1985)
- ♢ **Introduction to the Geometry of Foliations, Parts A,B**, [233]
  by Gilbert Hector and Ulrich Hirsch (1981)
- ♢ **Differential geometry of foliations**, [491]
  by Bruce Reinhart (1983)
- ♢ **Cohomology of infinite-dimensional Lie algebras**,
  by Dmitry Fuks (1984) (transl. 1986)
- ♢ **Embeddings and immersions**,
  by Masahisa Adachi (1984) (transl. 1993)
- ♢ **Partial differential relations**, [204]
  by Mikhael Gromov (1986)
- ♢ **Feuilletages: Etudes géométriques I, II**, [197]
  by Claude Godbillon (ms 1986) (appeared 1991)
- ♢ **Riemannian foliations**, [422]
  by Pierre Molino (1982) (transl. 1988)
- ♢ **Foliations on Riemannian manifolds**, [575]
  by Philippe Tondeur (1988)
- ♢ **Analysis on Foliated Spaces**, [425]
  by Calvin Moore and Claude Schochet (1988)
- ♢ **The structure of classical diffeomorphism groups**, [25]
  by Augustin Banyaga (1997)
- ♢ **Geometry of foliations**, [576]
  by Philippe Tondeur (1997)
- ♢ **Confoliations**, [143]
  by Yakov Eliashberg and William Thurston (1999)
- ♢ **Geometry of characteristic classes**, [429]
  by Shigeyuki Morita (1999) (transl. 2001)
- ♢ **Foliations I**, [71]
  by Alberto Candel and Larry Conlon (2000)
- ♢ **Introduction to the h-Principle**, [142]
  by Yakov Eliashberg and Nikolai Mishachev (2002)
- ♢ **Foliations II**, [72]
  by Alberto Candel and Larry Conlon (2003)

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